

# SUBSTANTIATION OF NEW DIAGNOSTIC PARAMETERS OF PIPELINE SYSTEMS EFFICIENCY

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## ABSTRACT

One of the main tasks of technical diagnostics of pipeline systems is to ensure their reliable and at the same time energy-efficient operation. In this work, we have searched for and developed the basis for constructing mathematical models of new informative parameters for diagnosing the technical condition and efficiency of pipeline systems. It is shown that the capacity of a pipeline decreases when it acquires an elliptical cross-sectional configuration. It is substantiated that the presence of small leaks in pipeline systems causes a loss of flow stability in the pipeline, the emergence of turbulent flow zones, which reduces the efficiency of the pipeline.

**KEYWORDS:** technical diagnostics, informative parameters, energy efficiency, pipeline systems, mathematical model

## INTRODUCTION

Technical condition of the pipeline system for transportation of oil, gas and petroleum products, compliance with its operating parameters within the established limits and efficiency of operation of a pipeline system, as a whole, directly affect the uninterrupted and reliable supply of carbohydrates to consumers.

The problem proposed for solution in this work, consists in an attempt to identify new diagnostic parameters, as well as in development of the basis for construction of mathematical models and procedures for evaluation of the influence of changes in these parameters on the technical condition and energy efficiency of pipeline systems. Such parameters, which are proposed to be used, include: geometrical parameters (wall thickness, diameter and cross-sectional shape of the pipeline), physical-chemical parameters of the transported product (viscosity, density, temperature), hydrodynamic parameters (pressure, flow rate, volumetric flow) and parameters associated with defects (leaks, cracks, deformations, corrosion or erosion damage).

Parameters, which change only slightly, while influencing the efficiency of pipeline systems operation, should include: small leaks, change in physical-chemical characteristics of the transported products, as well as a change in the cross-sectional geometry of the pipeline: its ovality or reduction of inner radius as a result of precipitation of heavy impurities and condensate in the pipeline cavity [1].

This kind of defects is determined by experimental methods. However, the methods of mathematical simulation of such phenomena are becoming ever wider

applied, in particular, because of the complexity of implementing the hardware methods, which is related to the inaccessibility of the surfaces for implementation of the contact methods of investigation and considerable scope of the required work.

Moreover, one of the main problems of technical diagnostics of pipeline systems is ensuring their reliable and at the same time energy efficient functioning. Thus, searching for new informative parameters of diagnostics of the technical state and efficiency of operation of the pipeline systems with development of the respective mathematical models is an urgent task.

## INVESTIGATION PROCEDURE

The problem of technical diagnostics and energy efficiency of pipeline system operation can be reduced to the problem of simulation of hydrocarbon flow in the pipeline with the available changes of its cross-sectional shape, presence of deposits on the pipeline inner surface, product leakage and change in the physical-chemical characteristics of the transported product proper, in order to develop the procedures and identify of the defects, and establish new informative parameters and limits of model application. When studying the technical condition of complex systems, which have been in operation for a long time, in particular, in the problems of their technical diagnostics, there are often cases, where the occurrence of emergency situations is caused by presence of minor disturbances and changes affecting the system [2].

Let us consider the problem of evaluation of the influence of a change in the cross-sectional geometrical characteristics of the pipeline and properties of the trans-

ported substance, on the technical condition and energy efficiency of a pipeline system. In the assumption that the pipe has a circular cross-section, and the fluid (oil or petroleum products) moves under the action of a stable pressure gradient along the pipe, the speed profile is determined by the Poiseuille formula [3, 4]:

$$w = \frac{i}{4\mu}(a^2 - r^2), \quad (1)$$

where  $w$  is the longitudinal flow speed of a viscous fluid;  $\mu$  is the fluid viscosity;  $a$  is the radius of the pipe, through which the fluid flows;  $i$  is the specific pressure gradient per a unit of pipe length;  $r$  is the radial coordinate. Here, the fluid speed profile is the paraboloid of revolution.

The volumetric fluid flow rate calculated by (1) is as follows:

$$Q = \int_0^a w 2\pi r dr = \frac{i\pi a^4}{8\mu}. \quad (2)$$

Here, it should be noted that the volume flow rate significantly depends on pipe radius and is proportional to the fourth power of its radius. Analyzing (1) and (2), we can draw the following conclusions: with the change of pipe radius that may occur during deposition of sediments on its internal wall, as was noted in [1], the volumetric flow rate reacts to it the most. Let  $a_1$  be the design radius of the pipeline,  $a_2$  — its radius after long-term service,  $a_1 > a_2$ , then in each point  $r = r_1$  the speed gradient  $\Delta w$  will be equal to:

$$\Delta w = \frac{i}{4\mu}(a_1^2 - a_2^2). \quad (3)$$

Here, a deficit of the capacity occurs, which can be assessed using (2) as follows:

$$\Delta Q = \frac{i\pi}{8\mu}(a_1^4 - a_2^4). \quad (4)$$

Using (4) transformation,  $\Delta Q$  value can be presented in the form of:

$$\Delta Q \approx \frac{i\pi}{8\mu} 4R^3 \Delta\delta, \quad (5)$$

where  $\Delta\delta$  is the change in the cross-sectional radius.

To compensate for such a deficit of the transported fluid, it is necessary to increase the relative pressure gradient, which can be determined from the following relationship:

$$i_1 = \frac{i(R - 4\Delta\delta)R^3}{(R - \Delta\delta)^4}. \quad (6)$$

It is obvious that  $i_1 > i$ . Note that increase in the pressure gradient reduces the energy efficiency of the system operation.

Another important moment and possible informative parameter is the transported substance viscosity. While at the initial moment of time the dynamic viscosity of the fluid is equal to  $\mu_1$  and at a certain moment of time it rose to the value of  $\mu_2$ , allowing for (1) and (2), we will have:

$$\begin{aligned} \Delta w &= \frac{i}{4\mu_1}(a^2 - r^2) - \frac{i}{4\mu_2}(a^2 - r^2) = \\ &= \frac{i}{4} \frac{\mu_2 - \mu_1}{\mu_2 \mu_1} (a^2 - r^2); \end{aligned} \quad (7)$$

$$\Delta Q = \frac{i\pi a^4}{8\mu_1} - \frac{i\pi a^4}{8\mu_2} = \frac{i\pi a^4}{8} \frac{\mu_2 - \mu_1}{\mu_2 \mu_1}. \quad (8)$$

To compensate for this ambiguity of the product, by analogy with (6), we can derive the value of a certain pressure gradient, required to compensate for the deficit:

$$\frac{i_1 \pi a^4}{8\mu_2} = \frac{i\pi a^4}{8\mu_1} - \frac{i\pi a^4}{8} \frac{\mu_2 - \mu_1}{\mu_2 \mu_1}, \quad (9)$$

$$\frac{i_1}{\mu_2} = \frac{i}{\mu_1} - i \frac{\mu_2 - \mu_1}{\mu_2 \mu_1}, \quad (10)$$

$$\frac{i_1}{\mu_2} = \frac{i(\mu_2 - \mu_2 + \mu_1)}{\mu_2 \mu_1} = \frac{i}{\mu_1}. \quad (11)$$

Finally, after carrying out the transformation, we get:

$$i_1 = i \frac{\mu_2}{\mu_1}. \quad (12)$$

It is obvious that  $i_1 > i$ , as  $\mu_2/\mu_1 > 1$ . Again, as we can see from the above, increase in the pressure gradient along the pipeline, leads to reduction in energy efficiency of the pipeline system.

Moreover, analyzing equations (1) and (2) and doing some transformations, we can obtain a generalized formula for speed gradient:

$$\Delta w = \frac{a^2 - r^2}{4\mu} \delta i - \frac{i}{4\mu^2} (a^2 - r^2) \delta \mu + \frac{i}{4\mu} 2a \delta a, \quad (13)$$

where  $\delta a$ ,  $\delta i$ ,  $\delta \mu$  are the variations of the respective values.

Similarly, we can obtain a generalized equation for capacity deficit:

$$\Delta Q = \frac{\pi a^4}{8\mu} \delta i - \frac{i\pi a^4}{8\mu^2} \delta \mu + \frac{i\pi 4a^3}{8\mu} \delta a, \quad (14)$$

Formulas (13) and (14) allow evaluation of insufficient amount (volume) of hydrocarbon flow in the cases of unambiguous determination of insufficient relative pressure gradient and change in the pipeline cross-section.

tional geometry as a result of deposition of technological substances on its walls and change in the fluid viscosity.

Cases also often occur, when investigation of the real geometry of pipeline systems reveals that the pipeline cross-section takes on an elliptical shape. This is due to the action of fluid-force factors (bending moment effect during shear, technological defects), which causes additional stresses in the pipeline material, and may lead to failure of its individual segments.

Analyzing the problem of fluid flow in the pipe, having the shape of an ellipse in its cross-section, we can come to the conclusion that formulas (1) and (2) can be written in the following form [3].

For the speed profile:

$$w(y, z) = A \left( 1 - \frac{y^2}{a^2} - \frac{z^2}{b^2} \right); \quad (15)$$

$$A = \frac{i}{2\mu} \frac{a^2 b^2}{a^2 + b^2}. \quad (16)$$

For volumetric flow rate:

$$Q = \iint_G w dy dz = \frac{\pi i}{4\mu} \frac{a^3 b^3}{a^2 + b^2}, \quad (17)$$

where  $a$  (major semi-axis) and  $b$  (minor semi-axis) are the constants of the ellipse curve  $\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$ , which simulates a deformed cross-section.

Let us consider function:

$$f(x, y) = \frac{x^3 y^3}{x^2 + y^2}, \quad (18)$$

as the function of two variables, and let us conduct its study for the extremum. From the system of equations (19) we can show that the above function has the following critical points:  $x = 0$ ;  $y = 0$ ;  $x = -e$ ;  $x = y$ :

$$\begin{cases} \frac{\partial f}{\partial x} = 0 = -\frac{x^3 y^3 2x}{(x^2 + y^2)^2} + \frac{3x^2 y^3}{x^2 + y^2} = 0, \\ \frac{\partial f}{\partial y} = 0 = -\frac{x^3 y^3 2y}{(x^2 + y^2)^2} + \frac{3x^3 y^2}{x^2 + y^2} = 0. \end{cases} \quad (19)$$

It is obvious that the first three conditions do not meet the physical conditions of the problem. Thus, the function has a separate extremum, which is reached at  $x = y$  ( $x, y, z$  are the coordinates of the studied pipeline segment: longitudinal, transverse horizontal and transverse vertical coordinates, respectively). This condition means that function (18) in such a case takes on an extreme value. By carrying out the respective mathematical transformations, we can prove that this is the main extremum, i.e. the maximum of the function.

From a practical point of view it means that the pipeline capacity is reduced with its cross-section tak-

ing on an elliptical shape, which results in the need to increase the relative pressure gradient and leads to lowering of the pipeline system energy efficiency [5].

The next diagnostic parameter can be derived upon detailed consideration of the viscous fluid flow in pipelines, containing pipe wall defects, in particular through-thickness holes, through which pipeline leakage occurs. In this case, the technical diagnostics problem can be presented in the form of the problem of simulation of a flow with leakage [6].

Investigations of the stability of hydrodynamic processes in the small leak area is important in two respects, influencing the energy efficiency: assessment of the amount of transported hydrocarbon loss; studying the flow structure in case of small leaks of different intensity, in terms of appearance of turbulent flow zones, which may lead to actual reduction in the pipeline effective diameter [7].

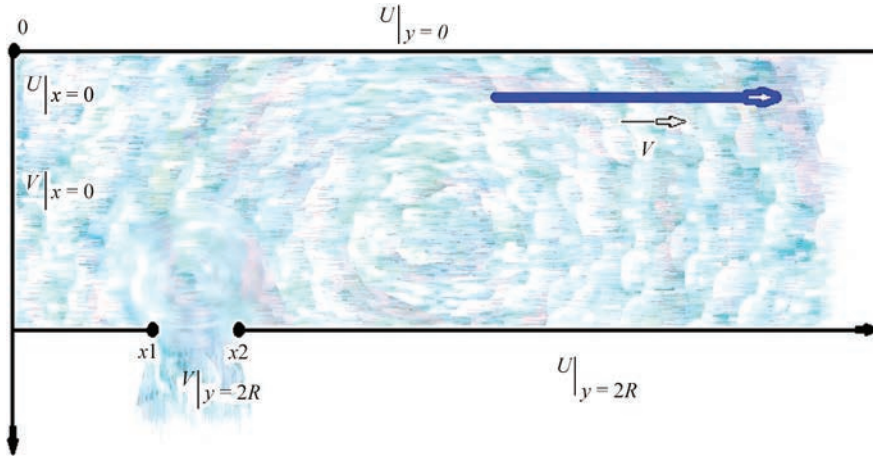
Mathematical simulation of the technological fluid flow in the pipeline was performed in case of leaks of different sizes through the pipeline wall. In order to implement it, a system of Navier–Stokes equations is numerically integrated, stability parameters of numerical schemes are studied, informative parameters are selected to determine the zones affected by the leak and the limits of the model application before the flow goes into the turbulent mode are established.

Product flow in the pipelines can be described using a system of Navier–Stokes equation, written in the cylindrical system of coordinates [8]. There is, however, one peculiarity of pipeline systems in terms of their geometry, in particular flow symmetry. The local nature of the small leak zone allows reducing the dimensionality of the problem and, in particular, believing that a two-dimensional flow of viscous fluid is considered in a channel with a wall, in which fluid leakage through the surface is present in the assumption that the flow becomes stationary. This assumption is valid in particular for quasistationary processes, when it is believed that the simulated flow characteristics change only little with time (Figure 1).

In such a case, the system of Navier–Stokes equations is written in a two-dimensional domain as follows:

$$\begin{cases} U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right), \\ U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right), \\ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \end{cases} \quad (20)$$

where  $U$  and  $V$  are the components of the speed vector in the rectangular Cartesian system of coordinates;  $\rho$



**Figure 1.** Flow schematic in a two-dimensional channel with leaks

is the density of the transported products;  $\nu$  is the coefficient of kinematic viscosity;  $p$  is the fluid pressure.

Boundary conditions are assigned as follows:

$$\begin{cases} U|_{x=0} = -\frac{ky^2}{4\mu} + \frac{kR_y}{2\mu}, \\ U|_{y=0} = U|_{y=2R} = 0, \\ V|_{x=0} = V|_{y=0} = 0, \\ V|_{y=2R} = \begin{cases} 0 & x < x_1, x > x_2, \\ V_{\text{leak}} & x \in [x_1; x_2], \end{cases} \end{cases} \quad (21)$$

where  $[x_1, x_2]$  is the leak zone;  $\mu$  is the dynamic viscosity of the transported products;  $R$  is the channel radius. For speed component  $U|_{x=0}$  it is assumed that it is calculated as in the known Poiseuille model [9], which describes a stationary flow of a viscous fluid in a pipe of a round cross-section.  $V_{\text{leak}}$  is the fluid leak rate through this domain. Boundary conditions (21) can be different, depending on how the fluid leak zones are located: if they are located on different channel boundaries, then for the component of speed  $V$  the speed values will be nonzero in different segments, both at  $y = 0$ , and at  $y = 2R$ . The method of solving the above problem is known [9]. A peculiarity of solving it is the presence of discontinuous boundary conditions (21) and absence of the correct boundary conditions for pressure.

Differentiating the first equation of system (20) with respect to variable  $x$ , and the second equation with respect to variable  $y$  and allowing for the third equation of system (20), we will get Poisson's equation for determination of pressure:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -2\rho \left( \frac{\partial V}{\partial x} \frac{\partial U}{\partial y} - \frac{\partial U}{\partial x} \frac{\partial V}{\partial y} \right). \quad (22)$$

The further solution scheme is as follows:

a) a certain initial approximation of pressure  $p_0(x, y)$  is assigned;

b) system (20) is solved with boundary conditions (21) with this distribution of  $p_0(x, y)$ ;

c) after defining speed components  $U$  and  $V$ , the right-hand parts of equation (22) are calculated;

d) equation (22) is calculated with the following boundary conditions:

$$p|_{\partial G} = p_0(x, y); \quad (23)$$

e) after deriving the new pressure distribution, the above algorithm returns to item (a).

This procedure should be repeated to achieve the convergence of the iterative process. System (20) with boundary conditions (21) is solved using absolutely convergent implicit schemes of the method of alternating directions [10], and equation (22) is solved by the method of successive over-relaxation. The convergence and stability of the above iteration method was proved in work [11].

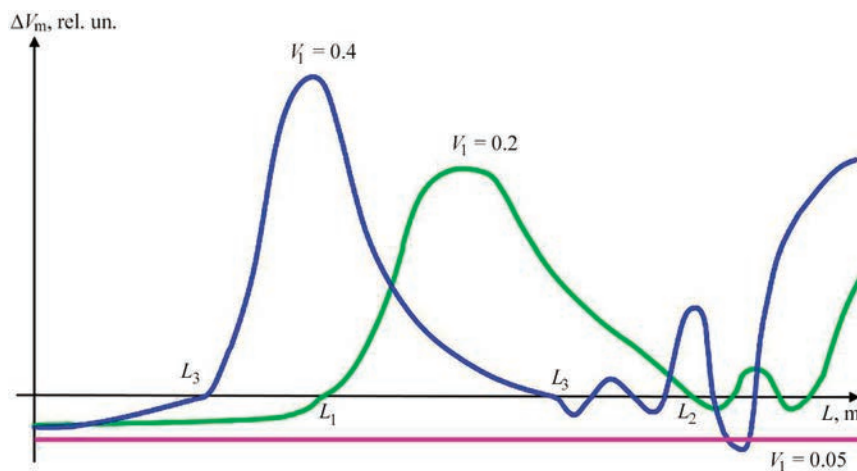
Initial approximation of pressure distribution was selected with the assumption of the existence of a linear pressure gradient along the channel, used to simulate a pipe with a leak:

$$p = p_0 - kx. \quad (24)$$

Using equation (24) for calculation of the speed field, we can establish the dependencies between the leak intensity and the change in flow configuration.

Simulation of a flow in a pipeline with defects through which the fluid leaks, is conducted for the following flow parameters, pipe geometry, properties of fluids and gases, and linear pressure gradient along the pipe length: average fluid speed in the pipeline is 2–8 m/s; characteristic small leak rate is up to 50 cm/s; dynamic viscosity of the fluid is 0.001 kg/m/s; kinematic viscosity is 0.000001 m<sup>2</sup>/s; pressure gradient characteristic  $K = 0.064$ – $0.096$ ; step along the longitudinal coordinate is 0.08 m; step along the transverse coordinate is 0.025 m, which corresponds to a pipeline 1.25 m in diameter with 50 control points along the transverse coordinate; number of steps along the





**Figure 2.** Dependence of the longitudinal component of speed  $\Delta V_m$  and the distance to the defect at different model values of the leak rate given in conditional units

longitudinal coordinate is 90000, which allows calculation of the speed field for a 7.2 km pipe with an 8 cm step.

Analyzing the behaviour of the longitudinal component of speed in the near-wall zone, we can note a regularity, which depends on the leak rate: the greater the leak rate, the faster is the speed field monotonicity violated on the leak side (Figure 2). Moreover, the following regularity was observed: monotonicity violation, which can be defined as the difference in the speed in two points of the grid, closest to the wall:

$$\Delta V_m = V(N) - V(N-1),$$

where  $N + 1$  is the number of points in the computational grid along the transverse coordinate.

Violation of the speed field monotonicity occurs by the following pattern: initially, the first monotonicity violation occurs, then the monotonicity is restored and its subsequent loss leads to loss of stability by the computational process, schematically shown in Figure 2.

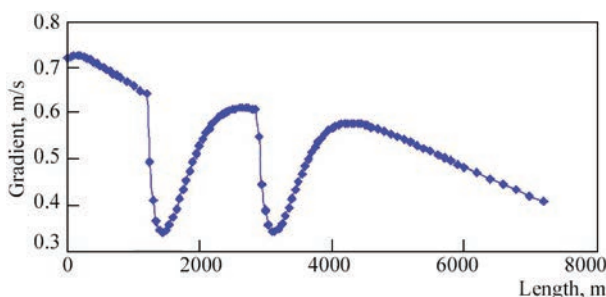
Points  $L_3$  and  $L_1$  can serve as a flow response to a small disturbance, they correspond to the minimal distance at which the effect of the disturbance is already noticeable, while points  $L_4$  and  $L_2$  are the points of the loss of stability of the difference scheme. In such a case points,  $L_3$  and  $L_1$  can be a diagnostic feature, while  $L_4$  and  $L_2$  cannot be such a feature. The content of the processes

occurring after these points can be as follows: either the stability of the computational procedure is lost, or the physical pattern of the flow changes, i.e. it moves from the laminar into a turbulent mode and further description of the flow requires application of other models. From the technical viewpoint such a behaviour is explained by the fact that with fluid slowing down along the pipe it is necessary to pump it up to ensure a certain pressure, flow speed and specified supply volumes, respectively. This leads to lowering of energy efficiency of the pipeline system. An important result, given in Figure 2, is the fact that at certain values of the leak rate ( $V = 0.05$ ), no loss of fluid monotonicity is observed at all. Thus, the higher the leak rate, the faster the flow reacts to it by changing the speed monotonicity in the near-wall zone.

After making the appropriate calculations, we can show that at leak rate values ( $V = 0.05 - 0.15$ ) the flow preserves its stability, i.e. both the stability of the hydrodynamic process, and the stability of the numerical scheme are in place. With increase of the leak rate, however, the pattern of distribution of the longitudinal component of the speed in the near-wall zone at different leak rates takes on a different character. In particular, the stability of the flow is lost, which is attributable to appearance of turbulent effects of the flow and a possible loss of stability of the difference method.

The developed model and numerical scheme of its implementation can be also used in diagnostics of small leaks, located at a certain distance from each other (Figure 3).

Proceeding from the results of numerical simulation of fluid flow through a channel with its leakage through the surface, the method of assessment of the coordinate of the leak point and its dependence on the leak rate was determined. It was confirmed that the problems of technical diagnostics of various-purpose systems in mathematical terms are the problems of studying the stability of the respective processes and numerical schemes of implementation of the models of such processes.



**Figure 3.** Dependence between the gradient of the longitudinal component of speed and presence of two leaks of different intensity located at a distance, leak coordinates  $x = 1.2$  and  $x = 2.88$  km

## CONCLUSIONS

As a result of the conducted studies, it was proposed to take into account new informative parameters (geometrical, physical-chemical, hydrodynamic, flaw detection) during assessment of the actual technical condition and efficiency of pipeline system operation, namely:

1. Change in the cross-section of the pipeline system due to deposits of the transported technological substances, changes in the dynamic viscosity of the transported substance, and specific pressure gradient lead to development of additional stresses in the pipeline material and even appearance of an elliptical configuration of the pipeline cross-section, which may result in destruction of individual segments of the pipeline.

2. Presence of small leaks in the pipeline wall, which form as a result of corrosion, material defects, mechanical damage, leads both to loss of the transported products, and to loss of flow stability in the pipeline, formation of zones of turbulent (unstable) flow, resulting in the risk of erosion wear of the pipeline wall and development of additional stresses in the pipeline material. In this case, an inverse problem can be also solved: the developed model of fluid flow in a pipeline and numerical scheme of its realization can be used during diagnostics of small leaks, located at a certain distance from each other.

3. Value of specific pressure gradient. Increase in the specific pressure gradient results in reduction of the system energy efficiency. This lowering is indicative of insufficient supply of the transported product. Both these factors lead to violation of the standard mode of pipeline operation, and, accordingly, to changes in the physical-chemical characteristics of pipeline material.

Further investigations should be aimed at improvement of the methods of small leak detection, development of new approaches to real-time monitoring the state of pipelines and integration of mathematical models into the systems of automatic control of pipeline systems. It will allow ensuring a more accurate assessment of the technical condition of pipeline systems and preventing significant energy losses.

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## CONFLICT OF INTEREST

The Authors declare no conflict of interest

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