REFINED MATHEMATICAL MODEL OF THE ELECTRIC ARC BURNING IN PLASMATRON WITH EXTERNAL CURRENT-CONDUCTING WIRE

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Mathematical model of electromagnetic processes has been refined for the arc burning in plasmatron with an anode wire. Comparative numerical analysis of electric, thermal and gas-dynamic characteristics of the arc plasma has been conducted by using a boundary layer approximation and the refined model of the electromagnetic processes. It is shown that the method of description of the said processes has a substantial effect on calculated distributions of characteristics of the arc plasma generated by the plasmatron under consideration.

Keywords: *plasmatron with anode wire, electric arc, mathematical model, plasma flow characteristics*

Study [1] puts forward a mathematical model to describe a turbulent flow of the electric arc plasma generated by the plasmatron with an external current-conducting anode wire. An assumption of smallness of radial component j_r of density of the electric current in the arc, compared with axial component j_z ($j_r << << j_z$), is made in this study within the framework of a boundary layer approximation. This approach fails to adequately describe electromagnetic characteristics of the arc burning in the plasmatron under consideration, as j_r has sufficiently high values, comparable with those of j_z, in the near-cathode region at exit from the plasma-shaping channel and in a region of cathode fixation of the arc. Therefore, the model of electromagnetic processes used in [1] needs to be refined, which is the purpose of this concise article.

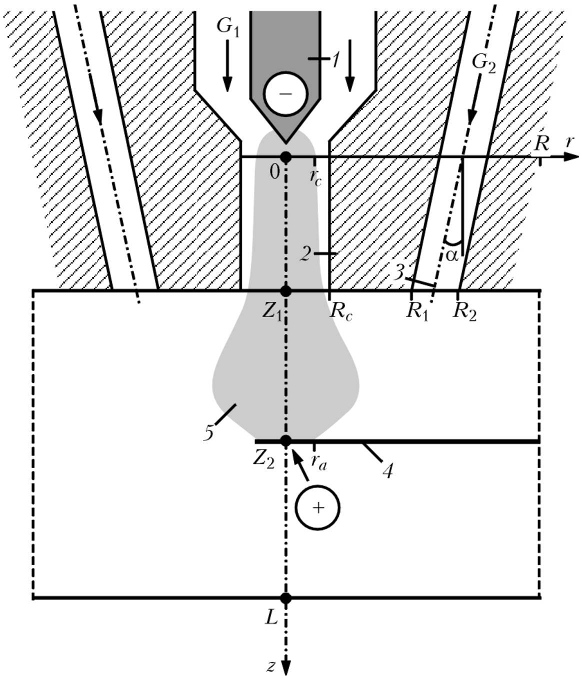

Figure 1. Schematic of the electric arc burning in plasmatron with external anode wire: *1* — refractory cathode; *2* — plasma-shaping nozzle; *3* — gas feeding channel; *4* — anode wire; *5* — arc column

Schematic of the DC arc being studied is shown in Figure 1. It burns in the plasma gas (argon) flow shaped by the plasmatron channel with radius R_c and length Z_1 between a refractory (tungsten) cathode and metal anode wire located after the exit section of a nozzle at distance Z_2 from the cathode tip. An exposed region of the arc discharge may be blown with a flow of a cold gas (argon, air) fed via an annular channel, which is located at angle α to the plasmatron axis and has internal, R_1, and external, R_2, radii. The calculation domain is limited by boundaries in radial, R, and axial, L, directions.

Gas-dynamic and thermal characteristics of the arc plasma generated by such a device can be described by a system of magnetic-gas-dynamic (MGD) equations in an approximation of a turbulent boundary layer [1]. For a more correct description of electromagnetic characteristics of the arc (with no assumption of smallness of the radial component of density of the electric current, compared with the axial one), we will use an equation of intensity of the magnetic field of the arc current [2]:

$$\frac{\partial}{\partial r}\left[\frac{1}{r\sigma}\frac{\partial(rH_\varphi)}{\partial r}\right]+\frac{\partial}{\partial z}\left[\frac{1}{\sigma}\frac{\partial H_\varphi}{\partial z}\right]=0, \quad (1)$$

where $\sigma[T(r, z)]$ is the temperature-dependent specific electric conductivity of the plasma; $H\varphi(r, z)$ is the azimuth component of intensity of the magnetic field related to the components of density of the electric current through the following equations [2]:

$$j_z=\frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi);\quad j_r=-\frac{\partial H_\varphi}{\partial z}. \quad (2)$$

Equations (1) and (2) can be complemented with a condition of conservation of the total current:

$$I = 2\pi \int_0^{R_\sigma(z)} j_z r dr, \quad (3)$$

where I is the arc current; and $R_\sigma(z)$ is the radius of the current-conducting region. Given that conductivity of the arc plasma is almost equal to zero outside this region, radius of the calculation domain can be used as an upper limit of integration in formula (3), i.e. it can be assumed that $R_\sigma(z) = R_c$ at $z \le Z_1$, and $R_\sigma(z) = R$ at $z > Z_1$ (see Figure 1). It should be noted that, as this model allows for both axial and radial components of density of the electric current, the Joulean source in energy equation [1] should be written down in the form of $(j_r^2 + j_z^2)/\sigma$.

Second-order differential equation (1) cannot be solved without setting the edge conditions for the entire contour of the calculation domain $\{0 \le r \le R_c$ at $0 \le z \le Z_1$, and $0 \le r \le R$ at $Z_1 < z \le Z_2\}$ (see Figure 1). To set the corresponding boundary conditions in the vicinity of the arc electrodes, i.e. at $z = 0$ and Z_2, we will use the experimental and calculation data from studies [3, 4] on densities of the current in the cathode and anode regions, assuming that

$$j_z(r, 0) = j_{0c} e^{-r/r_c}; \quad j_z(r, Z_2) = j_{0a} e^{-r^2/r_a^2}; \quad (4)$$

$$j_r(r, 0) = j_r(r, Z_2) = 0, \quad (5)$$

where j_{0c} and j_{0a} are the constants corresponding to the maximal values of density of the current in the cathode and anode regions; and r_c and r_a are the radii of the cathode and anode regions of fixation of the arc, respectively. In particular, at $I = 200$ A it is possible to use value $j_{0c} = 1.2 \cdot 10^8$ A/m^2 [3], and radius of the cathode region can be determined from total current conservation condition (3). An approximate radius of the anode wire is chosen as a radius of the anode region of arc fixation, and value j_{0a} is calculated from condition (3).

By substituting relationships (4) to the first equation of (2), we find:

$$H_\varphi(r, 0) = \frac{j_{0c} r_c^2}{r}\left[1 - e^{-r/r_c}\left(1 + \frac{r}{r_c}\right)\right];$$
$$H_\varphi(r, Z_2) = \frac{j_{0a} r_a^2}{2r}\left(1 - e^{-r^2/r_a^2}\right). \quad (6)$$

Then we set the boundary condition on axis of the system in the following form:

$$H_\varphi(0, z) = 0. \quad (7)$$

And assume the following at the external boundary of the calculation domain:

$$H_\varphi(R_c, z) = \frac{I}{2\pi R_c} \text{ at } 0 \le z \le Z_1;$$
$$H_\varphi(R, z) = \frac{I}{2\pi R_c} \text{ at } Z_1 < z \le Z_2. \quad (8)$$

Finally, at $z = Z_1$ and $R_c \le r \le R$, we have

$$H_\varphi(r, Z_1) = \frac{I}{2\pi r}. \quad (9)$$

Equations (1) and (2), together with boundary conditions (6) through (9), make up the basis of the refined mathematical model of electromagnetic processes for the arc burning in the plasmatron with an external current-conducting anode wire. These equations should be solved together with the MGD equations of a turbulent boundary layer [1], which determine spatial distributions of gas-dynamic and thermal characteristics of the arc plasma.

The complete system of equations was solved by the numerical finite difference method using global iterations with respect to intensity of the magnetic field (the procedure used to numerically solve the boundary layer equations is described in detail in [1]). The values of intensity of the magnetic field were updated at each new step of global iteration by numerically solving equation (1). This was done using the five-point implicit difference scheme by the sweep method [5]. The solution procedure was stopped when all characteristics of the plasma (at all points of the calculation domain) at two neighbouring iterations differed not more than by a preset low value of θ.

Analyse now the results of numerical modelling of electromagnetic, thermal and gas-dynamic characteristics of the arc plasma in the plasmatron under consideration by using approximation $j_r << j_z$ and the refined model of electromagnetic processes. The calculations were made for the plasmatron having a nozzle with length $Z_1 = 3$ mm and radius $R_c = 1.5$ mm, and an anode wire with a diameter of 1.4 mm located at a distance of 6.3 mm from the exit section of the nozzle ($Z_2 = 9.3$ mm). It was assumed that an external region of the plasma flow was blown with an argon flow going from the annular channel having radii $R_1 = 4.78$ mm and $R_2 = 7.22$ mm in the exit section, and inclined at angle $\alpha = 37.5°$ to the plasmatron symmetry axis (see Figure 1). The temperature of the cooled walls of the channels and ambient gas was assumed to be equal to 300 K. The following working mode was chosen for the plasmatron: arc current $I = 200$ A, plasma gas flow rate $G_1 = 1$ m^3/h, and blowing gas flow rate $G_2 = 20$ m^3/h.

Figure 2 shows the boundaries of regions $R_\gamma(z)$, within which the γ-fraction of the total arc current flows:

$$\frac{2\pi \int_0^{R_\gamma(z)} j_z r dr}{I} = \gamma = 0.1;\ 0.2;\ \ldots;\ 0.9, \quad (10)$$

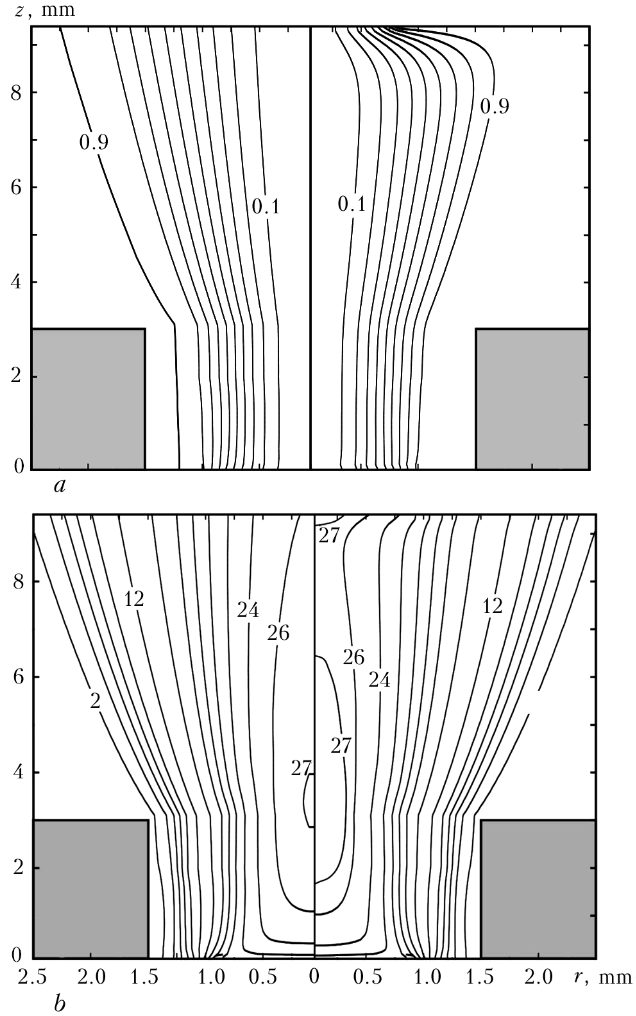

Figure 2. Isotherms of the electric current with a step of 0.1 (*a*) and isolines of temperature of the plasma with a step of T = 27, 26, 24 and then 2 kK (*b*) for the arc burning in plasmatron with an external anode wire for different methods of calculation of electromagnetic characteristics: *left* — approximation $j_r << j_z$; *right* — refined model

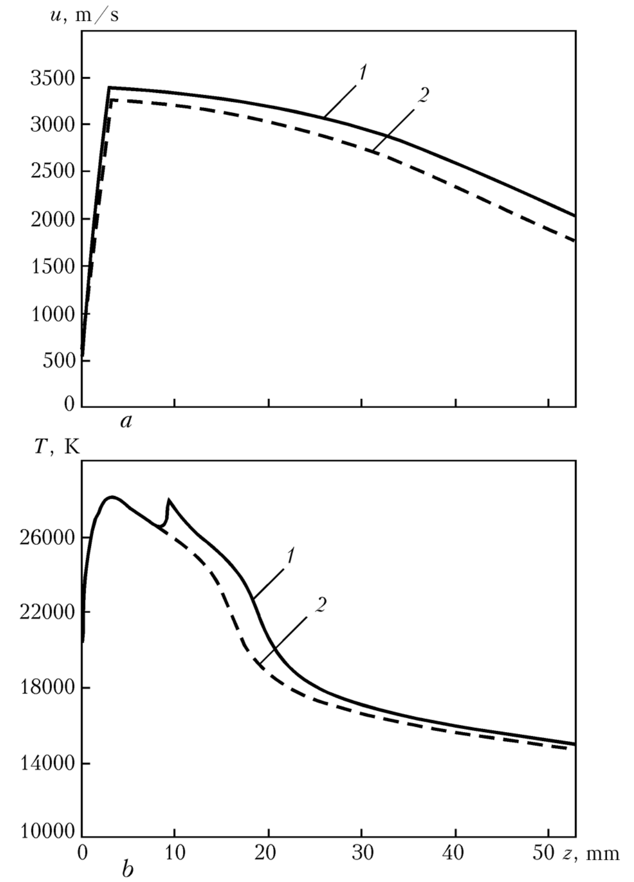

Figure 3. Distribution of axial velocity u (*a*) and temperature T of the arc plasma generated by the plasmatron with an external anode wire (*b*), determined by different methods of calculation of electromagnetic characteristics: *1* — refined model; *2* — boundary layer approximation ($j_r << j_z$)

which were calculated by using different calculation methods. As follows from this Figure, the use of approximation $j_r << j_z$ results in a current channel which gradually widens in an external region of the discharge (see Figure 2, *a*, *left*), and, in fact, coincides with a current-conducting region of the arc plasma (Figure 2, *b*, *left*). However, the solution of equation (1) showed qualitatively different results, especially in the exposed region of the arc, which is characterised by a substantial non-uniformity of the magnetic field along the length of the discharge and, accordingly, by higher values of the radial component of density of the electric current. As a result, the lines of the current, which diverge after leaving the plasma-shaping channel, converge towards the region of the anode fixation of the arc (Figure 2, *a*, *right*).

Figure 2, *b*, shows temperature fields corresponding to the chosen working mode of the plasmatron and calculated using different models of distribution of the current density in the arc. As can be seen from the isotherms shown in this Figure, differences in the calculated values of temperature turn out to be not as substantial as for the density of the electric current. However, we have to note an increase in the calculated temperature of the plasma near the region of the anode fixation of the arc when using the refined model of electromagnetic processes (see Figure 2, *b*, *right*). This is associated with the fact that the current channel determined within the framework of this model is markedly constricted in a region of the anode wire (see Figure 2, *a*, *right*), while the density of the current and, hence, Joulean heat sources is higher here and corresponds to a higher temperature. As to the gas-dynamic characteristics of the electric arc plasma in the plasmatron investigated, they hardly depend upon the method of calculation of its electromagnetic characteristics.

Consider now how thermal and gas-dynamic characteristics of the plasma flow change in a region of inertia movement of the plasma (after the anode wire) depending upon the model used to describe electromagnetic processes in the arc. The calculated distributions of axial values of the axial velocity and temperature of the plasma along the length of the jet are shown in Figure 3. With a variant of calculation involving the solution of equation (1), a higher temperature in the anode region of the arc leads to the fact that further on (in a no-current region of the flow) the temperature of the plasma will also be somewhat higher than with the calculation using simplifying approximation $j_r << j_z$. In the anode wire region the values of temperature at the discharge axis differ approximately by 14 %, further on they become almost identical, the difference being only 2–3 % (see Figure 3, *b*). The difference in the values of velocity of the plasma flow near the jet axis, on the contrary, grows with increase in the values of z: at z = 50 mm it amounts to 13 %.

CONCLUSIONS

1. The mathematical model of a turbulent flow of the arc plasma generated by the plasmatron with an external anode wire has been upgraded by refining the model of electromagnetic processes of the arc burning in such a device.

2. Spatial distribution of the electric arc current in the plasmatron with an external current-conducting anode wire has a non-uniform structure. The current lines diverge upon leaving the plasma-shaping channel, and converge with distance to the anode region, where the current density amounts to rather high values (over $1.5 \cdot 10^8$ A/m^2), thus providing an increased release of energy in the plasma. Ignoring the said peculiarities in numerical estimation of thermal and gas-dynamic characteristics of the generated plasma may lead to errors amounting to 14 % or more, depending upon the design parameters and working conditions of the plasmatron.

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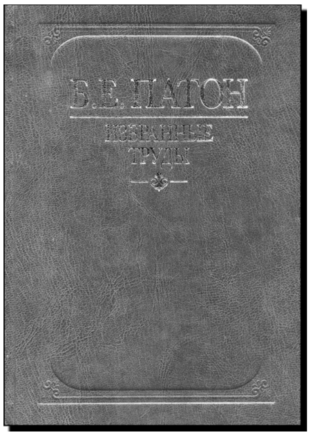
NEW BOOKS

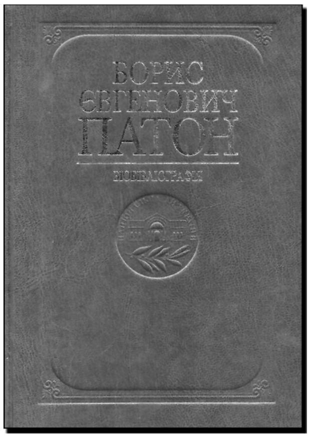
B.E. Paton. Selected works. — Ed. of the E.O. Paton Electric Welding Institute, NAS of Ukraine). — Kiev, 2008. — 894 p.



The collection is devoted to 90th anniversary of academician of NAS of Ukraine B.E. Paton — the prominent Ukrainian scientist in the field of welding, special electrometallurgy and materials science. The collection consists of seven sections which cover such directions of activity of B.E. Paton as fusion welding, pressure welding, arc welding metallurgy, special electrometallurgy, welded structures, space technologies and application of welding in medicine. Each section includes review of the works, the bibliography, and selection of the most important publications in which results of the works carried for the first time in the worl parctice are presented that had revolutionizing effect on development of leading bracnches of industry.

The work is intended for scientific and engineering-technical workers, professors, post-graduate students, and students of higher educational institutions.

B.E. Paton. — Introduction by I.K. Pokhodnya. — Kiev: Naukova Dumka, 2008. — 620 p.



The book describes main stages of life, scientific-organizational and public activity of the prominent scientist in the field of welding, metallurgy and technology of metals and materials science, the prominent public figure and the talented organizer of science — the academician of the National Academy of Sciences of Ukraine, Russian Academy of Sciences, professor, honored figure of science and technology of Ukr. SSR, twice Hero of Socialist Labor of the USSR, Hero of Ukraine, participant of Great Patriotic war, mitigation worker at Chernobyl NPP Boris E. Paton.

Index of the published works familiarizes a reader with scientific works of the scientist. The book is intended for scientific workers and all those who are interested in history of national science and technology.