allows achieving the optimum geometry, satisfactory formation and required reinforcement of the weld root.

Conducted studies showed that radiation of Nd:YAG laser of 4.4 kW power at welding speed of 16 m/h allows making in the butt joint a root weld with complete penetration and good formation of the back bead.

Optimum mode of welding the root welds in the joints of 25Kh2NMFA steel with U-shaped groove (Figure 3, *c, d*) is as follows: radiation power of 4 kW; welding speed of 16 m/h; focal distance of 200 mm; focal point deepening to 2 mm; gas flow rate: CO_2 — 20 l/min (to pool head), Ar — 10 l/min (pool tail part); feed rate of 1.2 mm wire — 38.4 m/h.

Thus, results of experiments on laser welding of root welds in the downhand position showed that with the appropriate fit-up and following the welding modes complete penetration of the weld root without defects (pores or cracks) with good formation of the back bead is ensured.

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OPTIMAL CONTROL OF FORMATION OF WELD REINFORCEMENT

V.V. DOLINENKO, T.G. SKUBA, V.A. KOLYADA and **E.V. SHAPOVALOV**
E.O. Paton Electric Welding Institute, NASU, Kiev, Ukraine

Method is proposed for development of the optimal system for automatic control of formation of the weld reinforcement with transportation lag in the feedback loop under MAG welding conditions. A dynamic model of formation of the weld reinforcement was developed to build the optimal controller. Mathematical modelling was performed by using the MATLAB software package. The developed control system provides a minimal duration of the process at preset limitations of dynamics of the adjustment actions.

Keywords: *MAG welding, dynamic model, weld reinforcement formation, mathematical modelling, optimal control system, transportation lag*

Achieving the optimal weld shape is one of the key tasks in fabrication of welded structures. This is explained by the fact that at the optimal shape of the weld reinforcement it is possible to decrease values of the stress concentration factor and improve performance of welded structures. Moreover, the required weld sizes allow minimising overuse of welding consumables under mass production conditions. Up to now, formation of the weld has been controlled by using an open circuit, through setting the welding process parameters. Peculiarities of design of the open systems to control formation of the welds, based on regression models, are considered in study [1]. Also, the weld shape can be controlled by using mechanical oscillations of the welding tool and magnetic control of the weld pool [2]. All open methods for control of the weld formation share one drawback, which is related to the absence of the mechanism to compensate for external disturbances, which affect a workpiece during the arc welding process and may lead to deviations of geometric parameters of the weld from the preset values. For example, such disturbances include ambient parameters, state of the surface and deviations of geometric parameters of a welding object. One of the methods to compensate for the external disturbances is to use the closed feedback systems for automatic control of the weld formation. A promising area of further advancement of the arc welding control systems is development and investigation of optimal and adaptive systems, the main advantages of which are considered in studies [3–5]. The necessity of applying the optimal control theory methods to welding is associated with high requirements for reliability and durability of welded structures [6].

The purpose of this study was to develop a system to control formation of the weld in MAG welding by using a laser TV sensor (LTS) in the feedback circuit to measure geometric parameters of the weld reinforcement bead.

Formalise the control problem, i.e. replace the control object by a mathematical model that describes essential peculiarities of the control problems and goals. The process of formation of the weld bead is a multidimensional connected control object, the behaviour of which can be described in first approximation by a system of first-order differential equations. In the state space, the object equations have the following forms:

$$\mathbf{x} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}\mathbf{u} + \boldsymbol{V}_0, \quad (1)$$

$$\mathbf{y} = \boldsymbol{C}\mathbf{x} + \boldsymbol{V}_o, \quad (2)$$

where $\mathbf{x}$ is the vector of state variables of the bead formation process $(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)^T$; $\mathbf{u}$ is the vector of control actions of the welding process $(\mathbf{u}_1, \mathbf{u}_2, ...,$

$\mathbf{u}_m)^T$; $\mathbf{y}$ is the vector of observations of geometric parameters of the bead $(\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_l)^T$; $\boldsymbol{A}$ is the matrix of states of the system measuring $n \times n$; $\boldsymbol{B}$ is the matrix of controls $(n \times m)$; $\boldsymbol{C}$ is the matrix of observations $(l \times n)$; $\boldsymbol{V}_0(t)$ is the matrix of coefficients of input disturbances; $V_o(t)$ is the matrix of coefficients of observation noises; and t is the time.

Matrices $\boldsymbol{V}_0(t)$ and $\boldsymbol{V}_o(t)$ are white noises with the following probability characteristics:

$$M[\mathbf{x}_0] = \bar{\mathbf{x}}_0 \text{ (average value)};$$

$$M[(\mathbf{x}_0 - \bar{\mathbf{x}}_0)(\mathbf{x}_0 - \bar{\mathbf{x}}_0)^T] = \boldsymbol{P}_0;$$

$$M[(\boldsymbol{V}_0(t) - \boldsymbol{V}_0^T(t'))] = \boldsymbol{Q}_0\delta(t - t');$$

$$M[(\boldsymbol{V}_o(t) - \boldsymbol{V}_o^T(t'))] = \boldsymbol{R}_0\delta(t - t');$$

$$M[\boldsymbol{V}_0(t)] = 0;\ \ M[\boldsymbol{V}_o(t)] = 0;\ \ M[(\boldsymbol{V}_0(t)\ \boldsymbol{V}_o^T(t'))] = 0,$$

where $\mathbf{x}_0 = \mathbf{x}(t_0)$; $\boldsymbol{Q}_0$ and $\boldsymbol{P}_0$ are the positively semi-definite matrices; $\boldsymbol{R}_0$ is the positively definite matrix; $\delta(t - t')$ is the Kronecker function; and t' is the time moment.

The optimality criterion, which has to be minimised, is set in the form of functional

$$J = M[\mathbf{x}^T(t_f)\ \boldsymbol{F}\mathbf{x}(t_f) + \int_0^{t_f} [\mathbf{x}^T(t)\boldsymbol{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\boldsymbol{R}\mathbf{u}(t)]dt], \tag{3}$$

where M is the mathematical expectation; $\boldsymbol{F}$ is the matrix of boundary conditions; and $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the matrices of weighting coefficients.

The optimal control problem is formulated as follows [7]: at preset object equations (1) and (2), control limitations $\mathbf{u}(t) \in \boldsymbol{U}_t$ and $\boldsymbol{U}_t \subseteq \Re^m$ (where $\Re^m$ is the m-dimensional linear space) and edge conditions $\mathbf{x}(0) = \mathbf{x}_0$ and $\mathbf{x}(t_f) = 0$, it is necessary to define such a control with feedback $\mathbf{u} = \mathbf{u}\{y(\tau), t_0 \le t \le t\}$, where $t_0 \le \tau \le t_f$, at which optimality criterion (3) would have a minimal value.

To solve the stated problem, represent the model of a control object (CO), which is the weld formation process, in the form of a connected system of dynamic links. Transition functions of the links should describe the transient processes at the CO output as precisely as possible. Welding experiments were designed and carried out to study the character of these processes and, as a result, generate the a priori information on dynamic characteristics of CO.

In the course of the experiments, the adjustment actions were formed as deviations in voltage U_a and welding current I_w, and welding speed v_w was maintained at a constant level. Welding was performed at the reverse polarity current in flat position in the atmosphere of a mixture of shielding gases (Ar + + 15 % CO_2). «Fronius TransPuls Synergic-5000» was used as an arc power source, and «Fronius VR 2000» — as a wire feed mechanism. The welding object was an 8 mm thick carbon steel plate. Electrode wire Sv-08G2S with a diameter of 1.2 mm was used for welding. The nominal welding parameters were as follows: $I_{w\,0} = 160$ A; $U_{a\,0} = 19$ V and $v_{w\,0} = 7$ mm/s. The amplitude of deviations of the adjustment actions for current was $\Delta I_{w\,max} = 15$ A, and for voltage — $\Delta U_{a\,max} = 2$ V.

It was found as a result of the experiments that stepwise variations of the adjustment signals lead to spurious oscillations of the bead surface and formation of undercuts. To prevent formation of defects in the welds, the rate of growth/fall of the adjustment signals for voltage was limited to 1 V/s, and for welding current — to 10 A/s. As shown by the experimental results, the process of the weld bead formation is characterised both by the dynamic behaviour and by the presence of two different transportation lags in formation of width and height of the bead.

Introduce the following designations: e and g — width and height of the weld bead; Δe and Δg — finite increments of width and height of the weld bead relative to nominal values e_0 and g_0; ΔU_a and ΔI_w — finite increments of the adjustment actions; and U_a and I_w — actual values of the adjustment actions. Therefore, the following equations are valid:

$$U_a(t) = U_{a\,0} + \Delta U_a(t);\quad I_w(t) = I_{w\,0} + \Delta I_w(t); \tag{4}$$

$$e(t) = e_0 + \Delta e(t);\quad g(t) = g_0 + \Delta g(t), \tag{5}$$

where $\Delta U_a(t) < \Delta U_{a\,max}$, and $\Delta I_w(t) < \Delta I_{w\,max}$.

Represent the weld bead formation model for steady-state welding conditions in the form of a static connected system linearised about the working point $(U_{a\,0}, I_{w\,0})$.

Write it down in the matrix form

$$\begin{bmatrix} \Delta e(t) \\ \Delta g(t) \end{bmatrix} = \begin{bmatrix} k_{11} & k_{21} \\ k_{12} & k_{22} \end{bmatrix} \begin{bmatrix} \Delta U_a(t) \\ \Delta I_w(t) \end{bmatrix}, \tag{6}$$

where k_{11}, k_{12}, k_{21} and k_{22} are the output-input gain factors, the values of which have to be determined.

Static and dynamic characteristics of the linearised weld formation process model were estimated from the welding experiments, which were conducted by using pulse adjustment actions U_a and I_w with a limited growth/fall rate. Note that in this case we actually determined the time constants of the power supply–weld pool–bead dynamic system. That is why the time constants have to be checked when changing the type of the welding equipment or technological process. Geometric parameters of the bead were measured after welding by using LTS [8] with a sampling increment of 1 mm. Mean values of $e_0 = 7.85$ mm and $g_0 = 2.2$ mm were subtracted from the obtained data arrays on geometric parameters of the beads, and then smoothed with a line filter of moving mean

$$N_O[i] = \frac{\sum_{i-3}^{i+3} N_I[k]}{7},$$

where N_O and N_I are the smoothed and initial data arrays, and i and k are the integers (indices of the arrays). To correctly estimate time constants of the four transit-time links that make up the model, the data arrays were shifted towards the electrode (to the right along axis x) over corresponding calculated distances of the transportation lags. The values of the transportation lags are determined as follows. It is a known fact that width and height of the bead form at the solidification front of metal of the weld pool in its middle and tailing portions [9]. Therefore, the transportation lags of measurements of height τ_g and width τ_e of the bead can be determined from the following formulae:

$$\tau_g = \frac{L_{TV} - L_g}{v_w}; \quad \tau_g = \frac{L_{TV} - L_e}{v_w}, \qquad (7)$$

where L_{TV} is the distance between the light trace of LTS and torch electrode, mm; L_g is the distance from the electrode to the end point of the weld pool tailing portion, mm; and L_e is the distance from the electrode to the middle point of the weld pool, mm.

A regression model was synthesised to calculate distances L_e and L_g. The model was developed by using a calculation experiment with the presented model of the process of propagation of heat in a semi-infinite body heated with a moving normal-rotary heat source [10]:

$$L_g = -0.69 + 0.041U_a + 0.0048I_w + 0.3v_w \text{ [cm]};$$

$$L_e = 0.08 + 0.004U_a + 0.0016I_w + 0.4v_w \text{ [cm]}.$$

Figure 1 shows the plots of variations in geometric parameters of the beads depending upon the linealy varying adjustment actions with reactions of transit-time elemens superimposed on them for comparison. Noteworthy is non-linear dynamics of variations in width and height of the bead under the effect of a welding current pulse, which shows up as decrease in duration of the output pulse due to the phase shift of its leading edge (Figure 1, I).

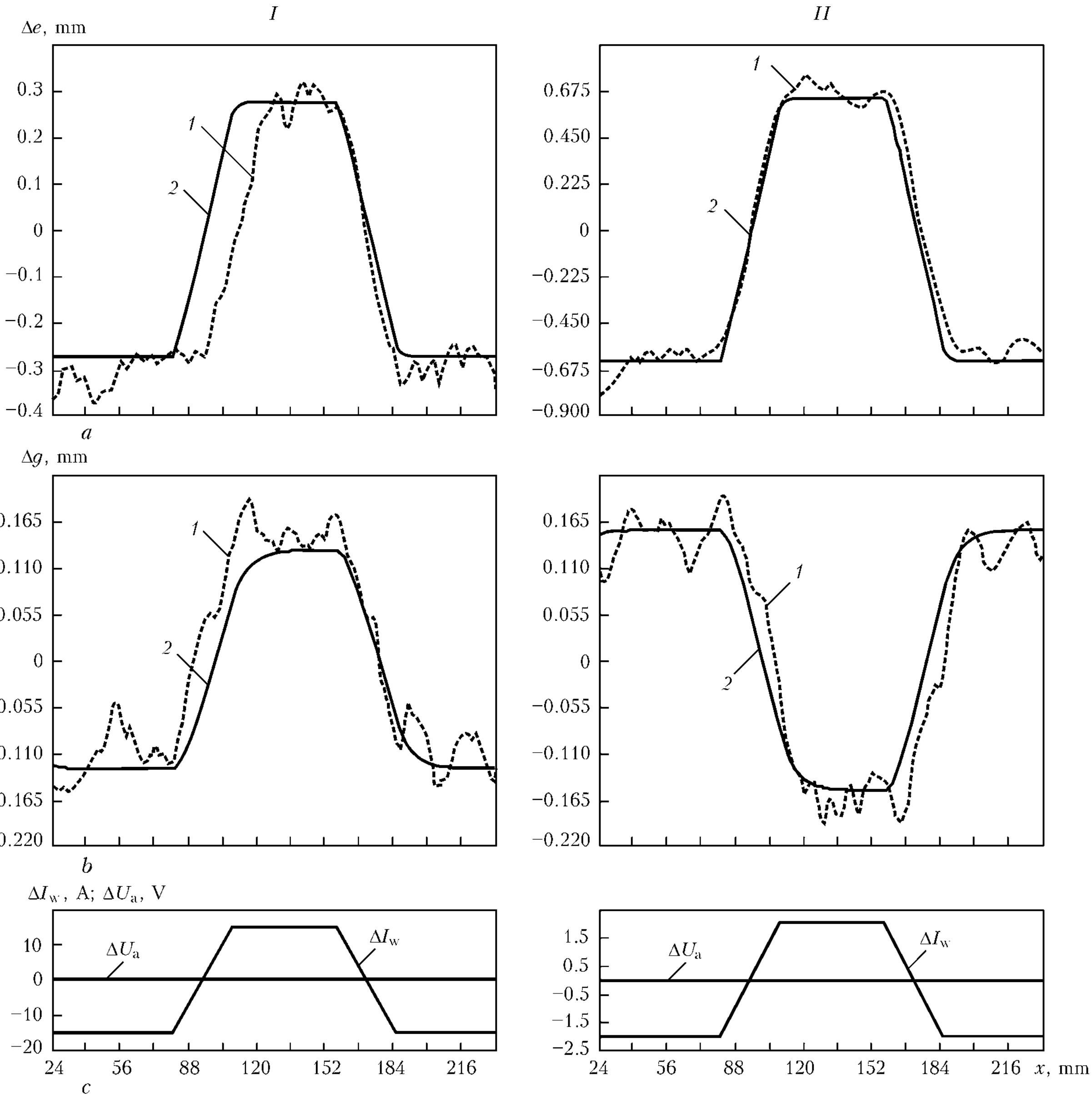

Figure 1. Results of identification of parameters of dynamic links of the weld reinforcement formation model for a pulse of welding current (I) and voltage (II); a, b — variations in width and height of the reinforcement bead, respectively; c — increments of welding current and voltage adjustment actions, respectively: 1 — reaction of dynamic links of the model; 2 — geometric parameters of the reinforcement bead measured with LTS

The dynamic control object is synthesised as follows. As it is necessary to limit dynamics of variations in adjustment actions U_a and I_w, the control vector is set in the form of time derivatives $\mathbf{u} = \left[\frac{dU_a}{dt}\ \frac{dI_w}{dt}\right]^T$. Then, to match the control and adjustment actions it is necessary to add an ideal integrating link to the CO composition. The output vector (observation vector) is set as $\mathbf{y} = [\Delta e(t)\Delta g(t)]^T$. The resulting output equation in the operator form is written down as follows:

$$\mathbf{y} = W(p)\mathbf{u}, \tag{8}$$

where $W(p) = W_3(p)W_2(p)W_1(p)$; p is the Laplace operator; $W_3(p)$ is the transfer function of an ideal link of the transportation lag; $W_2(p)$ is the transfer function of the first-order aperiodic link; and $W_1(p)$ is the transfer function of the ideal integrating link.

These transfer functions in the matrix form look like as follows:

$$W_1(p) = \begin{bmatrix} \frac{1}{p} & 0 \\ 0 & \frac{1}{p} \end{bmatrix}; \quad W_2(p) = \begin{bmatrix} \frac{k_{11}}{1 + T_{11}p} & \frac{k_{21}}{1 + T_{21}p} \\ \frac{k_{12}}{1 + T_{12}p} & \frac{k_{22}}{1 + T_{22}p} \end{bmatrix};$$

$$W_3(p) = \begin{bmatrix} e^{-\tau_e p} & 0 \\ 0 & e^{-\tau_g p} \end{bmatrix}.$$

After substitution of transfer functions $W_3(p)$, $W_2(p)$ and $W_1(p)$ in (8), the output equation will be written down as follows:

$$\mathbf{y} = \begin{bmatrix} \frac{k_{11}}{(1 + T_{11}p)p}e^{-\tau_e p} & \frac{k_{21}}{(1 + T_{21}p)p}e^{-\tau_e p} \\ \frac{k_{12}}{(1 + T_{12})p}e^{-\tau_g p} & \frac{k_{22}}{(1 + T_{22})p}e^{-\tau_g p} \end{bmatrix}\mathbf{u}. \tag{9}$$

The limitations have the following form:

$$\left|\frac{dU_a(e)}{dt} \le u_{\max 1}\right|, \quad \left|\frac{dI_w(e)}{dt} \le u_{\max 2}\right|, \tag{10}$$

where $u_{\max 1} \in \boldsymbol{U}$; $u_{\max 2} \in \boldsymbol{U}$; $\boldsymbol{U} \subseteq \Re^2_+$; $\Re^2_+$ is the two-dimensional space of non-negative numbers.

Figure 2 shows a structure chart of transfer function $W(p)$.

To make use of the known procedure of synthesis of the optimal automatic control system (ACS), exponential functions are replaced by rational polynomials. $e^{-\tau p}$ is approximated to a sufficient accuracy by Pade polynomial [11] of the 5th degree:

$$e^{-\tau p} = \frac{\tau^5p^5 - 30\tau^4p^4 + 420\tau^3p^3 - 3360\tau^2p^2 + 15120\tau p - 30240}{\tau^5p^5 + 30\tau^4p^4 + 420\tau^3p^3 + 3360\tau^2p^2 + 15120\tau p + 30240}. \tag{11}$$

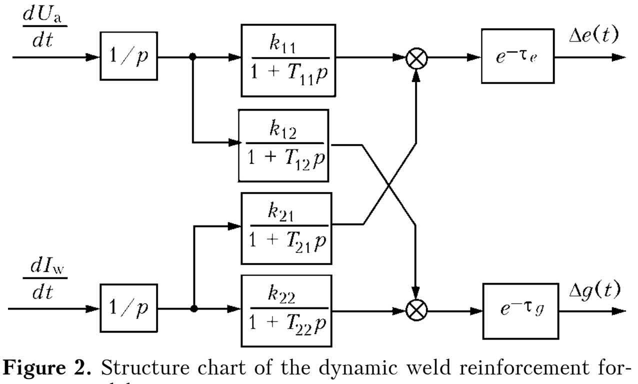

Figure 2. Structure chart of the dynamic weld reinforcement formation model

The resulting transfer function should be represented by a system in the state space. Mean values of the transportation lags, i.e. $\tau_e = 9.52$ s and $\tau_g = 8.38$ s, were determined based on the welding conditions (v_w = const) and preset distance $L_{TV} = 70$ mm. The calculations were made proceeding from an assumption that transportation lags τ_g and τ_e vary but insignificantly at the chosen range of the welding parameters. Hence, it follows that CO is stationary.

Control object matrices in the state spaces $\boldsymbol{A}$ (14 × × 14), $\boldsymbol{B}$ (14 × 2) and $\boldsymbol{C}$ (2 × 14), were calculated with the MATLAB package by using function *ss* [12].

Analysis of the obtained matrices shows that the quantity of lines of output matrix $\boldsymbol{C}$ is smaller than the dimension of matrix $\boldsymbol{A}$ that determines the state vector, for the restoration of which it is reasonable to use the Kalman–Bucy filter, i.e. optimal state observer.

Synthesis of the optimal control system for the weld bead formation is performed according to the procedure [7] based on the known principle of distribution or stochastic equivalence [13, 14]. It is used to solve the following interconnected problems: development of the deterministic optimal state controller and synthesis of the Kalman–Bucy filter. Development of the deterministic optimal controller is formulated as a problem of determination of optimal feedback control for object (1), (2) at optimality criterion (3)

$$\mathbf{u} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K}\hat{\mathbf{x}}, \tag{12}$$

where $\hat{\mathbf{x}}$ is the optimal estimate of the CO state, which is determined by using the optimal state observer, i.e. Kalman–Bucy filter; and $\boldsymbol{K}$ is the symmetric matrix determined from the Riccati matrix equation

$$\dot{\boldsymbol{K}} = -\boldsymbol{K}\boldsymbol{A} - \boldsymbol{A}^T\boldsymbol{K} + \boldsymbol{K}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K} - \boldsymbol{Q} \tag{13}$$

at boundary condition $\boldsymbol{K}(t_f) = \boldsymbol{F}$.

The required limitations on control are provided by a corresponding choice of matrix $\boldsymbol{R}$ and inclusion of the optimal controller of an auxiliary element into the system, which is described by function $u_{\text{out } k} = sat(u_{\text{in } k}, u_{\max k})$:

$$u_{\text{out } k} = \begin{cases} u_{\text{out } k}, & \text{if } u_{\text{in } k} < u_{\max k}, \\ u_{\max k}, & \text{if } u_{\text{in } k} \ge u_{\max k}, \end{cases} \quad \text{at } k = (1, 2, \ldots, m). \tag{14}$$

Synthesis of the Kalman–Bucy filter is made as follows. As noises of the welding process and observations are uncorrelated ($\boldsymbol{S}_0(t) \equiv 0$), the $\hat{\mathbf{x}}(t)$ estimate is unbiased and optimal if it satisfies equation

$$\dot{\hat{\mathbf{x}}} = \boldsymbol{A}\hat{\mathbf{x}} + \boldsymbol{B}\mathbf{u} + \boldsymbol{K}^0(\mathbf{y} - \boldsymbol{C}\hat{\mathbf{x}}); \quad \hat{\mathbf{x}}(t_0) = \bar{\mathbf{x}}_0 \tag{15}$$

with a matrix of gain factors $\boldsymbol{K}^0 = \boldsymbol{P}\boldsymbol{C}^T\mathbf{R}_0^{-1}$, where matrix $\boldsymbol{P}$ is the solution of the Riccati equation:

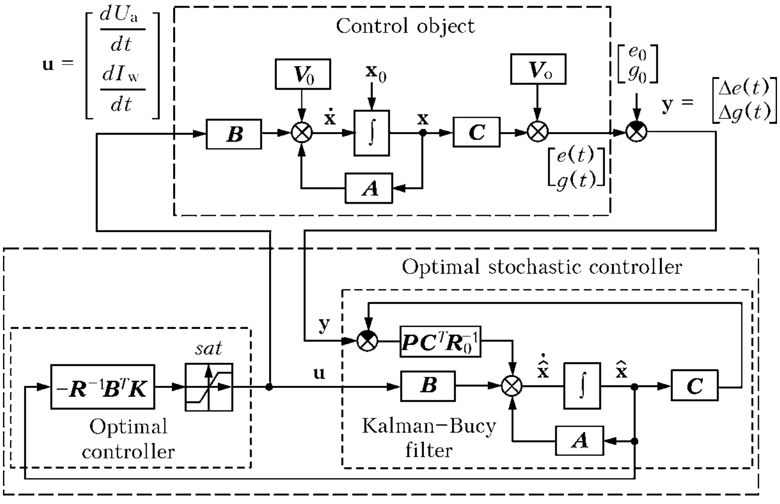

Figure 3. Structure chart of the optimal ACS model

$$\dot{\boldsymbol{P}} = \boldsymbol{AP} + \boldsymbol{PA}^T - \boldsymbol{PC}^T\mathbf{R}_0^{-1}\,\boldsymbol{CP} + \boldsymbol{Q}_0;\ \boldsymbol{P}(t_0) = \boldsymbol{P}_0. \quad (16)$$

Calculation of the stochastic optimal controller was made with the MATLAB package tools (by using function *lqry*). The resulting solution of the Riccati equation had the form of matrix $\boldsymbol{K}$ (14 × 14). The calculation was made by using matrices of weighting coefficients of the observation vector, $\boldsymbol{Q}$ (2 × 2), and control vector, $\boldsymbol{R}$ (2 × 2), in the following form:

$$\boldsymbol{Q} = \begin{bmatrix} 5 & 0 \\ 0 & 20 \end{bmatrix};\quad \boldsymbol{R} = \begin{bmatrix} 2 & 0 \\ 0 & 0{,}015 \end{bmatrix}.$$

Function *kalman* was used to calculate the Kalman–Bucy filter. Matrices of CO in the state space, matrices of coefficients of input disturbances, $\boldsymbol{V}_0$ (14 × 2), matrices of variation noises, $\boldsymbol{V}_{\rm o}$ (2 × 2), and covariance matrices of noises

$$\boldsymbol{Q}_0 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.2 \end{bmatrix};\quad \boldsymbol{R}_0 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.004 \end{bmatrix},$$

were set as input conditions.

Structure chart of the optimal ACS model (Figure 3) includes the CO model and model of the stochastic optimal controller, which consists of the optimal state controller, element of limitations of the control actions and optimal state controller, i.e. Kalman–Bucy filter. The optimal controller forms controls by way of derivative adjustment actions $U_{\rm a}$ and $I_{\rm w}$. Vector of the optimal observer state, $\hat{\mathbf{x}}$, is used as feedback signals. This vector is calculated on the basis of the a priori information on control object matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, as well as allowing for current values of the vector of controls and output vector $[\Delta e(t)\Delta g(t)]^T$.

Transient characteristics of the CO model were investigated. Figure 4 shows curves of the input and output signals in formation of pulse controls with a duration of 5 s and amplitude of 1 V/s and 10 A/s, respectively. These curves simulate the signals in hypothetic ACS with LTS, which forms a light trace on the workpiece surface at distance L_{TV} = 70 mm from the electrode axis. A change in values of geometric parameters of the bead with some transportation lags relative to the time point of feeding the control actions occurs in this case.

The transient and stationary processes in ACS were modelled by formation of the weld bead (Figures 5 and 6), considerable levels of noises of observation of the reinforcement width and height, equal to 0.2 and 0.05 mm, respectively, being simulated in this case. It was determined that fluctuations of output parameters insignificantly changed under the steady-state conditions, i.e. a change in fluctuations of the reinforcement width and height was no more than 0.05 and 0.02 mm, respectively. The modelling results al-

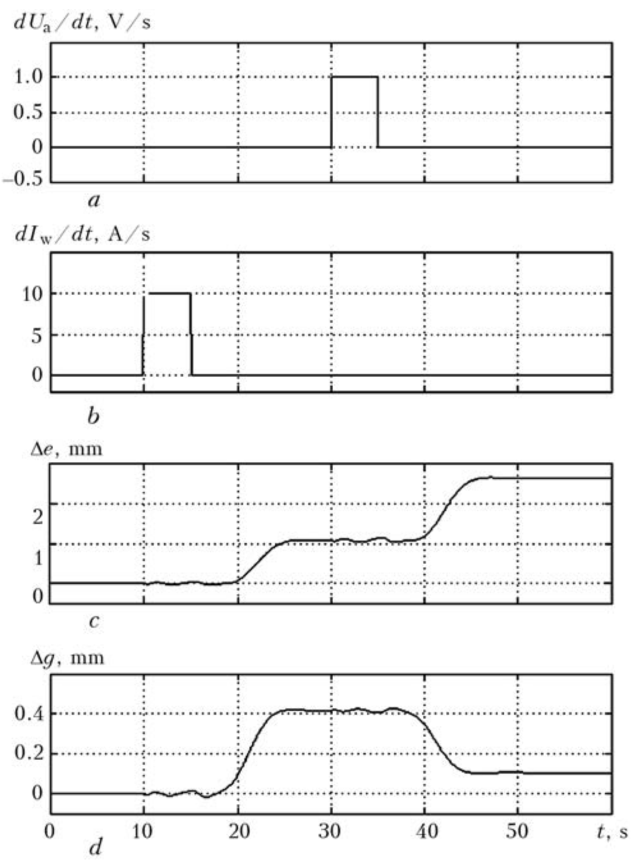

Figure 4. Transient characteristics of the dynamic control object model: *a*, *b* — voltage and welding current control actions, respectively; *c*, *d* — variations in weld width and height, respectively

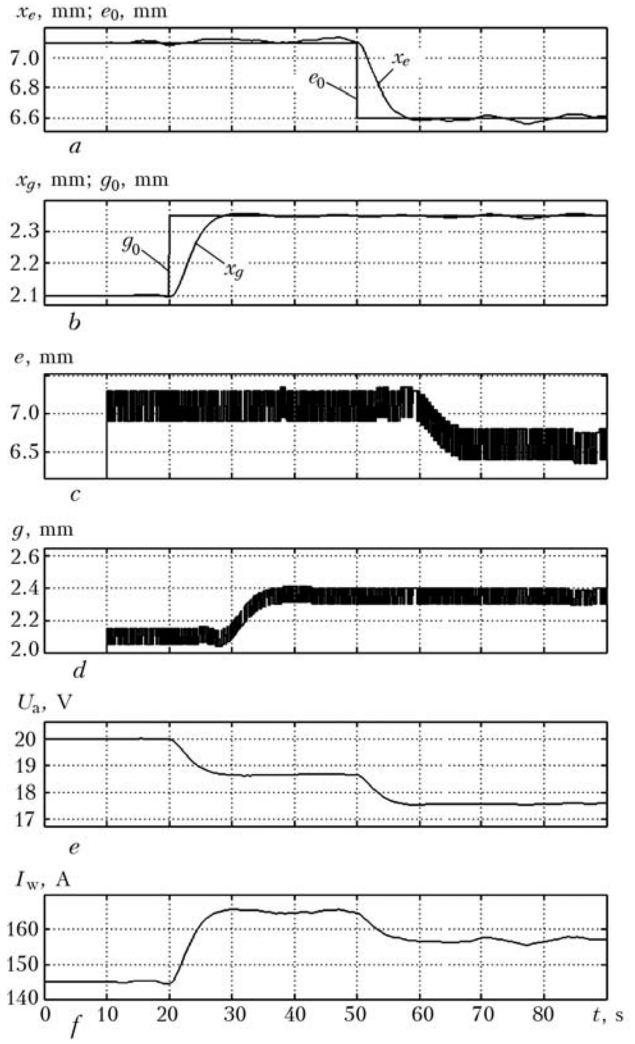

Figure 5. Curves of the transient process in optimal ACS with a change in master controls e_0 and g_0 at the 20th and 50th second: *a, b* — width x_e and height x_g of the bead at a current time point; *c, d* — signals of observation of width and height of the bead; *e, f* — adjustment actions

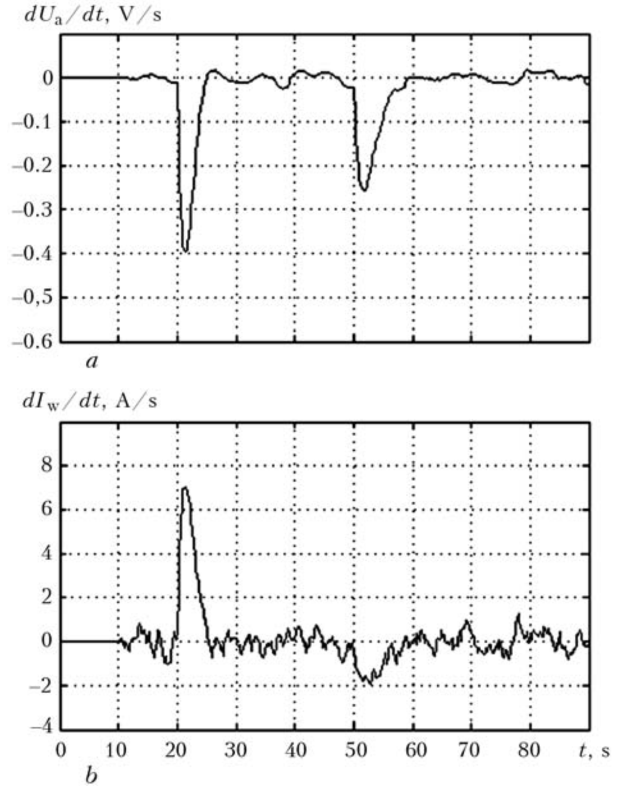

Figure 6. Curves of voltage (*a*) and welding current (*b*) signals in optimal ACS

low a conclusion that developed optimal stochastic ACS with a transportation lag in the feedback loop forms an acceptable path of control of the MAG welding process. Control of the weld reinforcement formation process described by the multidimensional dynamic system provides a minimal time of the transient process (no more than 8 s) at the absence of overcontrol. According to Figure 5, the control actions start simultaneously changing with a stepwise change in the master controls at time points of 20 and 50 s, this causing movement of CO state variables x_e and x_g. The paths of changes in the controlled parameters are of an aperiodic character.

The developed optimal controller provides limitations of dynamics of the adjustment actions. As follows from Figure 6, despite the transportation lag present in CO and a substantial level of noises, the control signals do not exceed the limitation levels of 1 V/s (for dU_a/dt) and 10 A/s (for dI_w/dt).

Therefore, it is likely that the approach proposed for synthesis of optimal ACS to control the MAG welding process can be further developed towards both refinement of the structure of the weld formation dynamic model and widening of the vector of control actions (e.g. adjustment of the welding speed) or observation vector (e.g. measurement of the joint gap).

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