MATHEMATICAL MODELLING OF STRESS-STRAIN STATE OF WELDED STRINGER PANELS FROM TITANIUM ALLOY VT20

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Improvement of methods for prediction of stress-strain state of welded joints in thin-sheet structures is a relevant problem. This, in particular, refers to stringer panels from VT20 titanium alloy which have high requirements on providing of high accuracy and strength at cyclic loads. Mathematical 3D modelling of stresses and deformations in small specimens (400 × 100–200 × 2.5 mm) with one stiffening rib and full size stringer panels (1100 × 550 × 2.5 mm) with four stiffening ribs was carried out under conditions of automatic non-consumable electrode welding by slot weld. Effect of preliminary elastic extension of plate and stiffening ribs on residual stress-strain state of panels was investigated. Computational investigations of stress-strain state for different variants of welding of the specimens showed that small width of panel specimens (100 mm) is not sufficient for determination of extension effect on residual stresses, and application of 200 mm width specimens is relevant for this purpose. The results of numerical calculations of stress-strain state for different variants of welding of stringer panels show principal possibility of performance at present time of such calculations in 3D problem statement for large welded structure with numerous welds, but at significant time consumption for calculation. 20 Ref., 3 Tables, 9 Figures.

Keywords: *welded stringer panels, titanium alloy VT20, welding stresses and deformations, method of preliminary elastic extension, mathematical modelling*

Prediction of welding deformations and stresses can be considered as an important factor of technological preparation of welded structure manufacture for selection of efficient method, mode and sequence of welding as well as geometry of stiffening ribs. Finally, all these factors are oriented on reduction of welding deformations in thin-sheet structures (as a rule the limits are set in specification).

Approximate engineering methods of calculation, based on knowledge of value of weld shrinkage function and application of methods of elasticity theory, obtained a wide distribution in predicting of stress-strain state of the welded panel structures. Analytical methods [1–3] or finite element method (FEM) [4] are used at that for problem solving. More general approaches of thermoplastic analysis in combination with FEM for 2D problem statement, i.e. at assumption of plane stressed state or plane deformation [5], are used for investigation purposes. Such a simplification of the model allows rapidly reducing the requirements to computational capabilities of the system and decreasing time for calculation.

Attempts of 3D modelling of panel welding with the help of FEM and methods of thermoplasticity theory are made at present time in the developed countries. The next researchers take the leading positions in development of specified problem. Thus, F. Boitout (ESI Group, France) carries out extensive computational investigation applicable to the welded structures, including in modelling of panel welding [6]. Investigations on optimization of welding process of light welded panels for minimization of the general deformations were performed by D. Camilleri and T. Gray (University of Strathclyde, Glasgow, Great Britain) [7]. Systematic computational investigations on the welded structures, including evaluation of the welding stresses and deformations, are made under the leadership of H. Murakawa (JWRI, Japan) [8–10]. P. Michaleris (Pennsylvania State University, USA) using 3D model studied deformations of shell-plate of the panel from welding of stiffening rib resulting in buckling failure [11]. Investigations carried out in recent time at the E.O. Paton Electric Welding Institute are also related with study of deformations and stresses in welding and thermal straightening of thin-sheet structures of panel type [12–14].

Mathematical 3D model using FEM and methods of thermoplasticity theory was developed for determination of stress-strain state of thin-sheet stringer panels in welding of longitudinal slot welds on the T-joints considering capabilities of current computer techniques and numerical methods of solving of boundary-value problem. It is used for performance of computational investigation of effect of different technological factors on level of residual stresses and deformations of welded stringer panels from VT20 alloy. This model allows also studying the torsion deformations caused by non-simultaneous performance of welds, besides the main types of deformations, i.e. transverse and longitudinal shrinkage, buckling and angular strains. 3D model in contrast to 2D model allows the effect of sequence of welding process on residual stresses and deformations to be determined.

Mathematical model of welding of small panel specimen (400 × 100 × 2.5 mm) with one stiffening rib was developed for preliminary evaluation of the stress-strain state of thin-sheet panels at different variants of technology of automatic welding. Table 1 gives reference data [15–18] on thermophysical and mechanical properties of VT20 alloy which were used in the calculation. Melting temperature of alloy makes $T_m = 1668 \pm$ ± 5 °C.

The developed computational model is based on numerical solution of the corresponding problems of thermoplasticity by means of tracing in time of development of temperature fields and connected with them kinetics of elastic-plastic deformations up to residual ones (complete smoothing of temperature) [5] in performance of the slot weld of panel specimen by moving heat source.

Modelling of the heat source is made in the following way. $\eta_s = 0.6$ coefficient of heating efficiency was set for automatic TIG welding by immersed arc based on comparison of calculation and experimental data by dimensions penetration zone. Rate of energy input in welding

$$q_{h.i} = \eta_s \frac{UI}{v_w}, \qquad (1)$$

where U is the voltage, V; I is the welding current, A; v_w is the welding speed, mm/s.

Power of heating W is traditionally distributed according to Gauss's law on the surface and depth of heated metal, i.e. in arbitrary point x, y, z of welded elements depending on welding speed v_w along the x-direction, i.e. in the Cartesian coordinates system in movement of the source on $z = z_0 = 0$ surface

$$W(x, y, z) = W_0 \exp \{- K_x[(-v_w t)^2 + y^2] - K_z z^2\}, \qquad (2)$$

where K_x is the coefficient of heat concentration on surface, i.e. in x- and y-directions, and K_z — on thickness (Figure 1). The latter parameters were selected from conditions $K_x = 12/B^2$ [19], where B is the width of weld pool. W_0 parameter is determined from energy balance:

$$W_0 = 2\pi \int_0^{\infty} \int_0^{\delta} \exp\,[-K_x\rho^2 - K_z z^2]\rho\, d\rho\, dz = q_s \qquad (3)$$

or

$$W_0 = \frac{2q_{\text{eff}}}{\frac{\pi}{K_x}\sqrt{\frac{\pi}{K_z}}},$$

where $\rho^2 = (-tv_w)^2 + y^2$; t is the time.

Heat rejection is performed on mechanism of heat conduction, i.e. determined by differential equation of heat conductance in the Cartesian coordinates system:

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + W(x, y, z) = c\gamma \frac{\partial T}{\partial t}. \qquad (4)$$

Symmetry condition is fulfilled at $y = 0$ (fusion line)

$$\frac{\partial T}{\partial y} = 0 \qquad (5)$$

on panel surfaces $z = 0$, $z = \delta$

$$\frac{\partial T}{\partial z} = \pm\, \alpha_y(T - T_0), \qquad (6)$$

where T_0 is the initial uniform temperature, equal ambient temperature; α_y is the coefficient of surface heat emission.

Stress fields and deformation fields as well as displacement vector U_j ($j = x, y, z$) in each point (x, y, z) in instant of time t can be calculated knowing temperature field $T(x, y, z, t)$ in different points (x, y, z) of the welded elements of panel. A solution is found by means of successive tracing of development of elastic-plastic state in time t starting from initial $t = 0$, $\sigma_{ij} = 0$, $\varepsilon_{ij} = 0$ and $T = T_0$. The solution on each tracing step t is based on a previous solution step in moment $t - \Delta t$, where Δt — tracing step by time t. Temperature field $T(x, y, z, t)$ is the disturbing factor.

Theory of Prandtl–Reiss plastic flow associated by Mises flow rule is used for solution of the thermoplasticity problem. Linearized problem on each step of tracing was solved using FEM. Physical non-linearity is realized in iteration way. At that, dependence of physical-mechanical properties of VT20 alloy on temperature (see Table 1) was considered in the computational model. Packet of computer programs «Weldpredictions» [20] developed at the E.O.

Table 1. Thermophysical and mechanical properties of VT20 titanium alloy depending on temperature at $\nu = 0.35$

T, °C	λ, W/(mm·°C)	c, J/(kg·deg)	$E \cdot 10^{-5}$, MPa	$\alpha \cdot 10^{6}$, 1/°C	σ_y, MPa
20	8	0.549	1.20	8	850
100	8.80	0.565	1.20	8.20	726
200	10.20	0.587	1.20	9.10	601
300	10.90	0.628	1.14	9.80	478
400	12.20	0.670	1.04	9.90	478
500	13.80	0.712	0.96	10.20	478
600	15.10	0.755	0.84	10.40	478
700	15.17	0.782	0.75	10.50	478
800	15.17	0.795	0.70	10.60	361
900	15.17	0.809	0.60	10.70	244
1000	15.02	0.808	0.50	10.70	128
1156	15.40	0.830	0.50	10.70	32
1157	15.30	0.770	0.54	9.50	32
1200	16.70	0.791	0.50	9.70	21
1400	18.30	0.827	0.50	10.99	21
1600	21.20	0.910	0.50	12.56	21
1800	23.70	0.997	0.50	12.56	21
1944	25.30	1.065	0.50	12.56	21
2000	25.30	1.230	0.50	12.56	21

Note. E — modulus of elasticity of material; ν — Poisson's ratio; α — coefficient of temperature expansion; λ — heat conduction coefficient; c — specific heat capacity; σ_y — yield strength.

Paton Electric Welding Institute is used for problem solution.

Computational investigation of stress-strain state of small specimens of panel (400 × 100 × × 2.5 and 400 × 200 × 2.5 mm) with one stiffening rib at different variants of preliminary extension in welding was carried out based on the developed mathematical model. Data in Figure 2 show that the preliminary extension of the panels has little effect on level of residual stresses of small specimen 100 mm width. The main influence is provided by rigid fixing of the specimens in welding. This results in significant reduction of maximum residual stresses in comparison with welding in a free state. A conclusion can be made analysis of calculation data that 100 mm width of small specimens of panels is not sufficient for evaluation of influence efficiency of the preliminary extension on residual stresses in the full-sized panels.

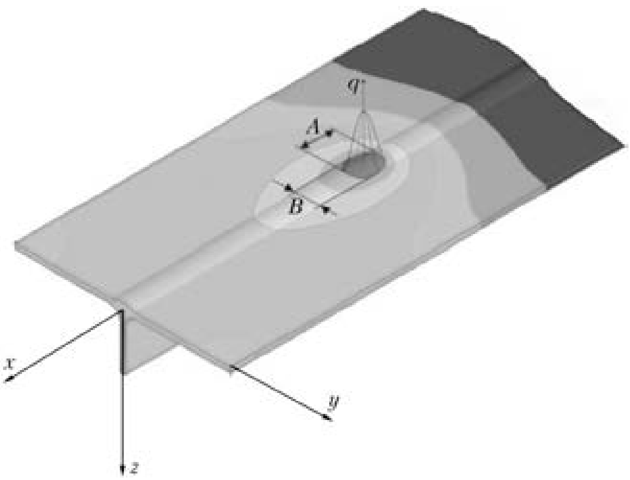

Figure 1. Scheme of heat source in welding of panel specimen by slot weld: A — length of weld pool; B — width of weld pool; q — Gaussian distribution of heat power

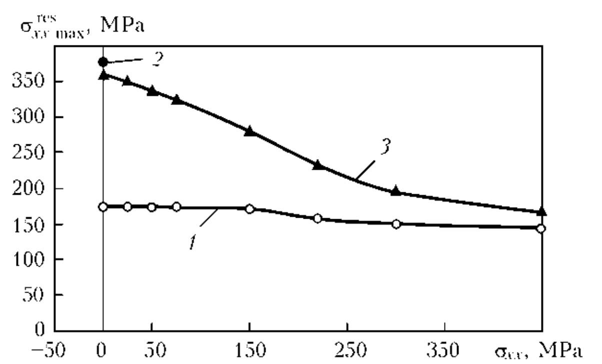

Figure 2. Dependence of maximum residual longitudinal stresses $\sigma^{res}_{xx\,max}$ on preliminary extension σ_{xx}: *1* — welding in a free state; *2*, *3* — welding with extension at B = 200 and 100 mm, respectively

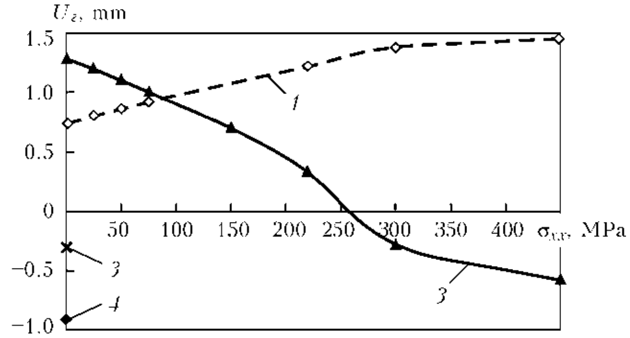

Figure 3. Dependence of residual buckling U_z on preliminary extension σ_{xx}: *1*, *2* — welding at extension B = 200 and 100 mm, respectively; *3*, *4* — welding in a free state at B = 200 and 100 mm, respectively

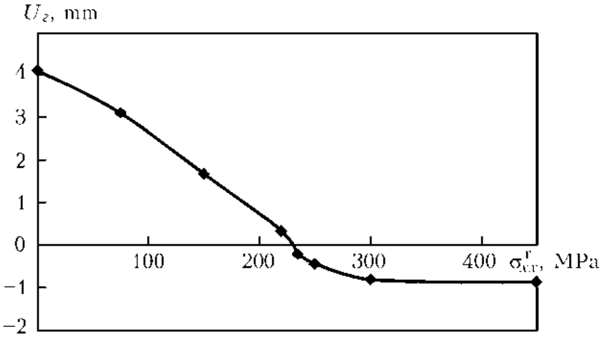

Figure 5. Dependence of residual buckling U_z on preliminary extension of rib σ^{r}_{xx} at plate extension σ^{p}_{xx} = 220 MPa

Effect of preliminary extension on maximum residual stresses (see Figure 2) significantly increases and principle change of influence of preliminary extension on residual buckling of the panel takes place (Figure 3) if width of panel specimen achieves 200 mm. At that, the preliminary extension of sheet and rib of panel specimen on 250 MPa level is an optimum from point of view of minimization of residual stresses (2 times reduction) and buckling deformations (close to zero).

Figure 4 shows good matching of experimental (curve *1*) and calculation (curve *2*) data in case of welding without preliminary elastic extension except for weld zone. In the latter experimental determination of residual stresses by means of deformation measuring is not accurate enough since can be performed only from one side of the specimen where the stiffening rib is absent. Calculation data (curve *5*) are confirmed by experimental ones (curve *2*) in welding of the specimen with 220 MPa preliminary elastic extension of plate.

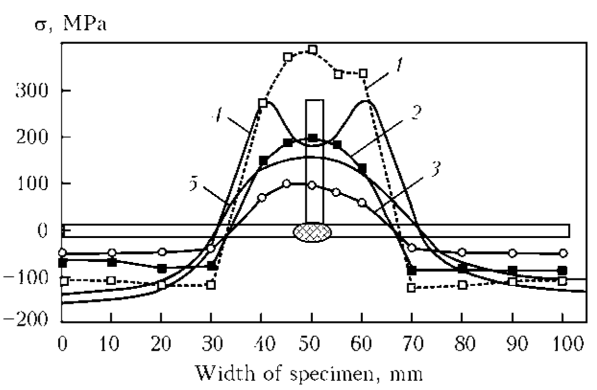

Figure 4. Experiment (*1–3*) and calculation (*4*, *5*) data on residual longitudinal stresses in welding of specimen in fixture with extension (*1*), under conditions of preliminary longitudinal elastic extension at σ = 220 and 450 MPa, respectively (*2*, *3*), in fixture without extension (*4*) and under conditions of preliminary longitudinal elastic extension at σ = 220 MPa (*5*)

Computational experiments were carried out for determination of effect of the preliminary extension of stiffening rib on maximum residual longitudinal stresses in the specimen of stringer panel (400 × 200 × 2.5 mm) at 220 MPa plate preliminary extension. Preliminary extension of the stiffening rib has significant influence on residual buckling of specimen of stringer panel, as can be seen from calculation data (Figure 5). Minimum buckling is provided at value of stiffening rib preliminary extension close to preliminary extension of the plate ($\sigma^{r}_{xx} = \sigma^{p}_{xx} = 220$ MPa). Thus, if preliminary extension of the plate is 220 MPa, extension of the stiffening rib is reasonable to be at the same level or somewhat higher (230–240 MPa) for reduction of buckling deformations.

Calculation results (Figure 6) show small dependence of maximum residual longitudinal stresses of specimen of the stringer panel (400 × × 200 × 2.5 mm) on preliminary stiffening rib extension if plate preliminary extension makes 220 MPa. It should be noted that obtained calculation data on welding of stringer panels have experimental confirmation.

A model for determination of stress-strain state in welding of 1100 × 550 × 2.5 mm stringer panel with four stiffening ribs was created based

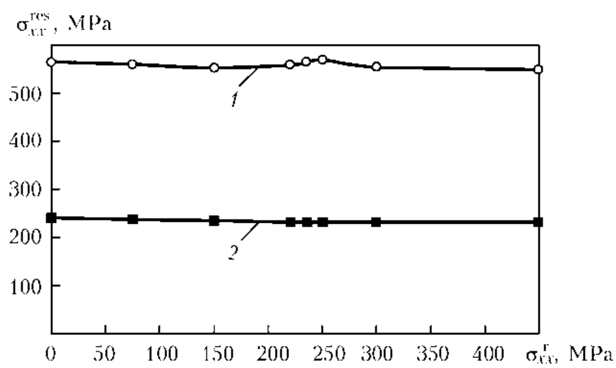

Figure 6. Dependence of maximum residual longitudinal stresses σ^{res}_{xx} on preliminary extension of rib σ^{r}_{xx} after welding (*1*) and extension removal (*2*) at plate extension σ^{p}_{xx} = = 220 MPa

on the developed mathematical model for small specimen of stringer panel.

Computational algorithm of determination of stress-strain state during stringer panel welding includes three main successive steps of modelling:

1. Setting of preliminary elastic extension σ by means of fixing of one transverse edge of plate and ribs and displacement of the second edge in longitudinal direction by $\Delta = 1100\sigma/E$ (mm) value, fixing of transverse edges from sheet plane as well as fixing of longitudinal edges of ribs in direction from plate plain;

2. Modelling of welding (by slot weld) of four longitudinal ribs in set sequence and specified welding direction using movable heat source;

3. Release of welded panel by means of removal of all fixtures.

The first and third steps of modelling were carried out per one time step and they are, respectively, connected with loading and unloading of model of stringer panel in the elastic field.

In a process of movement of welding heat source the finite elements are fastened between themselves in the range of penetration zone, simulating fusion of stiffening rib and plate between themselves.

Calculation of stress-strain state of the stringer panels was carried out at different preliminary extension and set asymmetrical sequence of welding (Figure 7). The welds were performed in sequence from one end of the panel to another and only in one direction for showing maximum effect of non-simultaneous weld performance on distribution of the residual stresses and deformations.

Table 2 demonstrates the results of calculation of maximum residual longitudinal stresses and residual buckling of the panel (1100 × 550 × × 2.5 mm) depending on general preliminary extension.

Time of calculation of the stress-stain state in welding of stringer panels is sufficiently long. Calculation of one variant took on average 15 days using modern personal computer with quad-core processor. This work presents the results for

Table 2. Maximum residual longitudinal stresses and residual buckling of panel depending on general preliminary extension

Plate/rib		$\sigma^{res}_{xx\,max}$, MPa		σ^{res}_z, mm
σ, MPa	Δ, mm	After welding	After fixing removal	
0/0	0/0	550	316	15.1
80/80	0.73/0.73	550	313	12.5
220/220	2.02/2.02	550	223	7.6
300/300	2.75/2.75	550	195	5.2

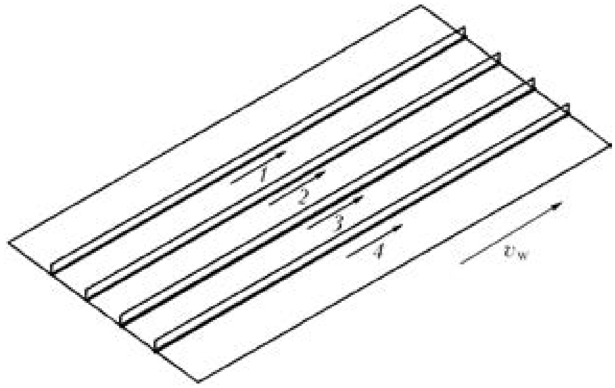

Figure 7. Scheme of weld performance sequence in modelling of welding of stringer panel

only four variants of stringer panel welding. It allows showing the principal possibility of performance at present time of such calculations in 3D problem statement for sufficiently large welded structures with numerous welds.

Distribution of the residual longitudinal stresses σ_{xx} (Figure 8, *a*) on the surface of stringer panel after welding of all welds (immersed-arc welding at $v_w = 13$ m/h, $I = 250$ A, $U = 11.5$ V) under conditions of fixing with preliminary extension ($\sigma_{xx} = 220$ MPa) was obtained on the second step of modelling. Maximum longitudinal extension residual stresses of 450–550 MPa level appear in average section of the panel, and they achieve up to 650 MPa in zone of end effects of welding.

Significant redistribution of the longitudinal residual stresses (Figure 8, *b*) takes place after removal of fixing from welding panel. Maximum

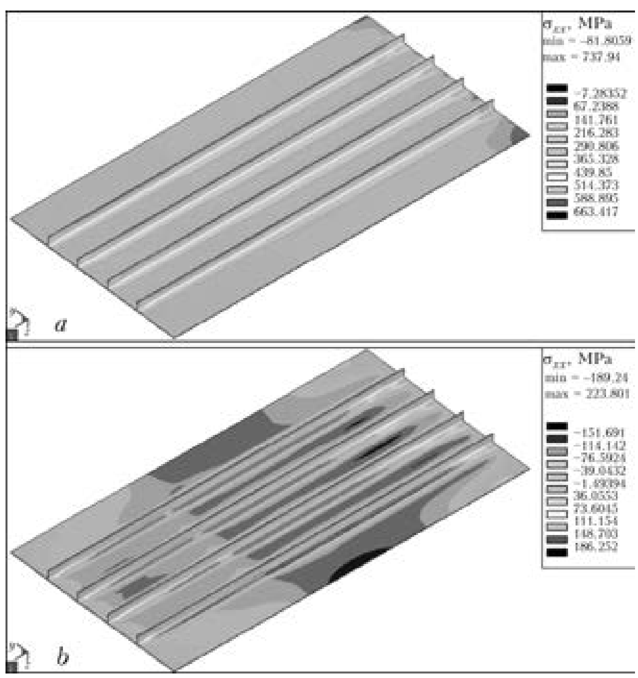

Figure 8. Distribution of residual longitudinal stresses σ_{xx} after welding and complete cooling of all four ribs (*a*) and after fixing removal (*b*)

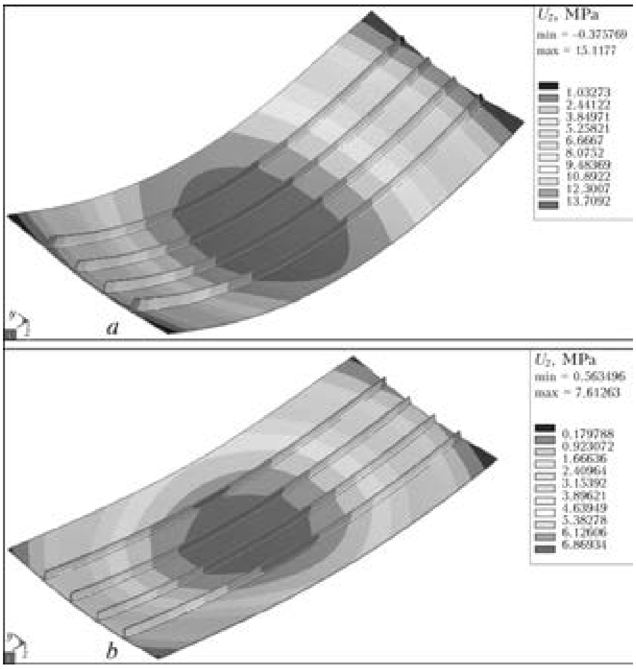

Figure 9. Residual buckling of stringer panel during welding in fixing without preliminary extension ($U_{z\,max}$ = 15.1 mm) (*a*) and with preliminary extension σ_{xx} = 220 MPa ($U_{z\,max}$ = = 7.6 mm) (*b*)

longitudinal residual extension stresses reduce up to 220 MPa in the average section of panel. Noticeable asymmetry of distribution of residual stresses related with non-simultaneous weld performance is observed. The values of residual stresses in the average section of panel in zone of last weld approximately 40 MPa higher than in zone of previous welds for which maximum longitudinal extension stresses do not exceed 180 MPa.

Residual deformations also appear on the third step of modelling. In considered case of welding with σ_{xx} = 220 MPa preliminary extension the maximum residual buckling deformations achieve 7.6 mm (Figure 9, *b*) that is significantly lower of 15.1 mm value of buckling for welding of fixed stringer panel without preliminary extension (Figure 9, *a*). Nevertheless, the values of buckling for cases of welding with σ_{xx} = 220 and 300 MPa preliminary extension appeared to be higher as a result of calculation then were predicted (calculation of small specimens of panels). Besides, insignificant asymmetry of distribution of residual buckling with displacement of the maximum towards the last weld related with non-simultaneous performance of weld runs is also observed. At that, any noticeable torsion deformations of stringer panel are absent. This can be seen on similar values of displacements from angle plain of the panel. Therefore, fixing of stringer panel in welding for elimination of displacements from plane and ends of the panel in longitudinal direction is efficient for prevention of residual torsion deformations.

Residual buckling deformations in a form of violation in plain were measured as pointers of longitudinal buckling on line of the ribs W_{r1}, W_{r2}, W_{r3}, W_{r4}, and as pointers of buckling of the end and average cross sections W_{c1}, W_{c2}, W_{c3}, W_{c4}, and as a torsion angle α of the cross sections relatively to each other. The results of measurements are given in Table 3.

Comparison of calculation and experiment data (see Table 3) shows that the method provides a character of distribution of residual deformations similar to measurement results, however, the values of longitudinal buckling are overestimated (maximum calculation buckling is 7.6 mm and experimental one makes 5.0 mm). Besides, the calculation angle α is sufficiently lower than its experimental values (calculation α = 0.2° and experimental — 1.6°). Probably, the input data (parameters of preliminary elastic extension) in calculation performance had insufficiently correspondence to the same parameters at experiment, either the real conditions of welding of stringer panels were not completely considered by developed mathematical model and the latter requires further improvement and experimental check.

In conclusion note that the results of modelling of stress-strain state for different variants of

Table 3. Experiment and calculation data on residual buckling deformations and torsion deformation of welded stringer panels

Number of panel	W_{r1}, mm	W_{r2}, mm	W_{r3}, mm	W_{r4}, mm	W_{c1}, mm	W_{c2}, mm	W_{c3}, mm	α, deg
1	2.5	3.5	2.0	2.0	3.0	5.0	3.5	2.5
2	2.0	2.5	3.0	3.0	4.0	4.5	4.0	3.0
3	1.5	2.0	2.0	1.5	2.5	3.5	2.0	1.0
4	1.5	2.0	1.5	1.5	1.5	2.0	2.0	1.0
5	1.0	1.0	1.5	1.5	1.5	1.5	3.5	1.5
6	0.5	1.5	1.0	0.5	1.0	2.0	1.0	0.5
Average experimental data	1.5	2.1	1.8	1.7	2.3	3.1	2.7	1.6
Calculation data	6.9	7.4	7.5	7.1	5.0	7.6	5.5	0.2

welding small specimens of stringer panels 400 × × 100 × 2.5 mm and 400 × 200 × 2.5 mm with one stiffening rib showed:

• 100 mm width of the panel specimen is not enough for determination of efficiency of extension influence on residual stresses;

• significant effect of the preliminary extension on maximum residual stresses and buckling of the panel is noticeable at 200 mm width of the specimen and, at that, preliminary extension of sheet and rib of the panel specimen on 250 MPa level is optimum from point of view of minimization of residual stresses (2 times reduction) and buckling deformations (close to zero);

• preliminary rib extension has significant influence on residual buckling of stringer panel specimen of 400 × 200 × 2.5 mm size as well as on the fact that the minimum buckling is provided at rib extension close to plate preliminary extension ($\sigma^{r}_{xx} = \sigma^{p}_{xx}$ = 220 MPa), i.e. at plate preliminary extension on 220 MPa level for the rib extension at the same level or somewhat above (230–240 MPa) is reasonable to be provided buckling deformation reduction. At that, maximum residual longitudinal stresses of stringer panel specimen have small dependence on preliminary rib extension.

The results obtained in modelling of stress-strain state for different variants of welding of 1100 × 550 × 2.5 mm size stringer panels with four stiffening ribs showed:

• principle possibility of performance at present time of such calculations in 3D problem statement for sufficiently large welded structure with numerous welds, but considering long enough time for calculation;

• high efficiency of preliminary elastic extension method on reduction of a level of residual stresses and buckling deformations of the stringer panels;

• efficiency of fixing of stringer panel in welding for elimination of displacements from plane and ends of the panel in longitudinal direction to prevent appearance of residual torsion deformations.

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