INSTABILITY OF MODE IN CIRCUIT WITH CAPACITY AND ELECTRIC ARC SUPPLIED BY DIRECT CURRENT SOURCE

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Stability and appearance of oscillations in electric arc current were investigated for specific conditions of DC electric circuit. It is indicated that circuit with arc has always parallel to it capacity, formed by unit own capacities. The electric arc as element of electric circuit is described using generalized model, which considers thermal delay of electric arc and does not limit its static volt-ampere characteristic. Considered is an effect of parameters of electric arc on transfer process, conditions of appearance of continuous and divergent oscillations are obtained. Principle and equivalent schemes of investigated circuit are presented. Dissipative properties of oscillation system are characterized with the help of coefficient of circuit attenuation, specific damping as well as coefficient of energy loss in the system. Suppression frequency, pass band, frequency of own oscillations and dynamic resistance of the circuit are determined. Resistive damping of oscillations is studied, desirable value of damping resistor is determined and results of calculations and modelling are illustrated. Received results can be used in designing and adjustment of new power sources for welding and related technologies as well as estimation of damping and stabilizing of operating power sources. 8 Ref., 4 Figures.

Keywords: *arc welding, electric and plasma arc, stability of process, transfer processes, oscillation damping, calculation and modelling*

Stability of electric arc under specific conditions in electric circuit was multiply investigated [1–8]. It is important to note that circuit with the arc has always parallel to it capacity, formed by unit own capacities. At that output capacity, for example, in welding inverters, is used as a fixing (damping) circuit or for noise reduction [7]. These capacities achieve 0.01 μF and even make several microfarads [1, 4, 7] considering capacity of trigger elements of arc and mains. Now let's study stable and unstable state of electric arc with capacity and its effect on electric circuit in more details.

Electric arc as element of the electric circuit is described in present study using generalized model, which considers thermal delay of the electric arc and does not limit its static volt-ampere characteristic (VAC). As a result, differential resistance R_{dif0} and series to it small stray inductance L, shunted by active resistance $R1$, were simulated in the scheme for investigation of stability of uncontrolled nonlinear resistance — electric arc. In this case investigated electric circuit (Figure 1) is formed by parallel connection of ideal current source, element with input arc resistance $Z_{\text{a}}(p)$, resistance R_i and capacity C.

Presence of resistor R_i takes into account all types of losses in the system, i.e. final (even if sufficiently high), internal (output) resistance of current source as well as effect of external circuits.

Characteristic of impedor is its input (or internal) resistance:

$$Z_{\text{in}}(p) = Z(p) = \frac{k_1 + k_2 p}{1 + T_1 p + T_2^2 p^2}, \qquad (1)$$

where $k_1 = R_{\text{dif0}}$; $k_2 = \theta R_{\text{st0}}$; $T_1 = \theta + R_{\text{dif0}}C$; $T_2^2 = \theta R_{\text{st0}}C$; θ is the constant of arc time

If $p = j\omega$ is included in equation (1), then it describes frequency transfer function of the circuit at $0 \le \omega \le \infty$:

$$Z(j\omega) = \frac{k_1 + k_2 j\omega}{1 + T_1 j\omega + T_2^2 (j\omega)^2},$$

modulus of which and phase are, respectively, the following

$$|Z(j\omega)\} = \frac{\sqrt{k_1^2 + k_2^2\omega^2}}{\sqrt{(1 - T_2^2\omega^2) + T_1^2\omega^2}},$$

$$\psi(\omega) + \arg Z(j\omega) + \operatorname{arctg} \frac{k_2\omega}{k_1} - \operatorname{arctg} \frac{T_1\omega}{1 - T_2^2\omega^2}.$$

Formula (1) is good to be represented as follows:

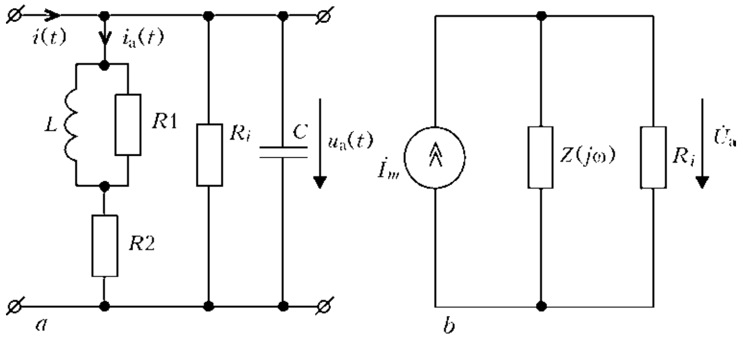

Figure 1. Principle (a) and equivalent scheme of studied circuit (b): $L = \theta(R_{st0} - R_{dif0})$; $R_1 = R_{st0} - R_{dif0}$; R_{st0}, $R_2 = R_{dif0}$ – static and differential resistance of arc in selected working point I_0

$$Z(p) = \frac{k_1 + k_2 p}{1 + 2\xi T p + T^2 p^2} = \frac{k_1 + k_2 p}{1 + \dfrac{2\xi p}{\omega_0} + \dfrac{p^2}{\omega_0^2}},$$

where

$$T = T_2 + \sqrt{\theta R_{st0} C};$$

$\xi = T_1 / 2T = \dfrac{1}{2}\left(\sqrt{\dfrac{\theta}{R_{st0} C}} + \sqrt{\dfrac{C}{\theta R_{st0}}}\, R_{dif0}\right)$ is the damping parameter; $\omega_0 = 1/\sqrt{\theta R_{st0} C}$ is the frequency of own oscillations of the system;

$\alpha = \xi\omega_0$.

The next representation is used for description of impedor:

$$Z(j\omega) = r(\omega) + jx(\omega),$$

where

$$r(\omega) = \mathrm{Re}Z(j\omega) = \frac{k_1(1 - T_2^2\omega^2) + k_2 T_1 \omega^2}{(1 - T_2^2)\omega^2 + T_1^2\omega^2};$$

$$x(\omega) = \mathrm{Im}Z(j\omega) = \frac{k_2\omega(1 - T_2^2\omega^2) - k_1 T_1 \omega}{(1 - T_2^2\omega^2) = T_1^2\omega^2}.$$

Given operator resistance has the only zero at $p = -R_{dif0}/R_{st0}\theta$ and two poles in points with coordinates

$$P_{1,\,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} = -\frac{1}{2}\left(\frac{1}{R_{st0}C} + \frac{R_{dif0}}{R_{st0}\theta}\right) \pm$$

$$\pm\, j\sqrt{\frac{1}{\theta R_{st0} C} - \frac{1}{4}\left(\frac{1}{R_{st0}C} + \frac{R_{dif0}}{R_{st0}\theta}\right)^2},$$

which can be complexly-associated as well as real ones depending on relationship between ω_0 and α.

Figure 2 shows the family of amplitude- and phase-frequency characteristics of the circuit with following parameters: $\theta = 1$ μs; $R_{st0} = 1.25$ Ohm, $R_i = \infty$, $R_{dif0} = -0.49$ Ohm.

Generation of divergent own oscillations in the electric circuit with losses (see Figure 1, a) is possible only, if the circuit, in addition to passive elements R, L, C, includes also active ones, transmitting part of energy from external sources into the circuit. A widespread model of such active element is a resistor with negative resistance. Studied circuit becomes unstable (will be self-excited), if present in it negative resistance $R_{dif0} > R_{dif0\,cr}$ (condition of self-excitation lies in complete compensation of circuit loss).

Capacity C can be selected in such a way that the impedor would be damped in desired pass band, that eliminates effect of resonance peak. Cutoff frequency of the impedor can be made respectively large in selection of capacity C, providing value of damping parameter around 0.7.

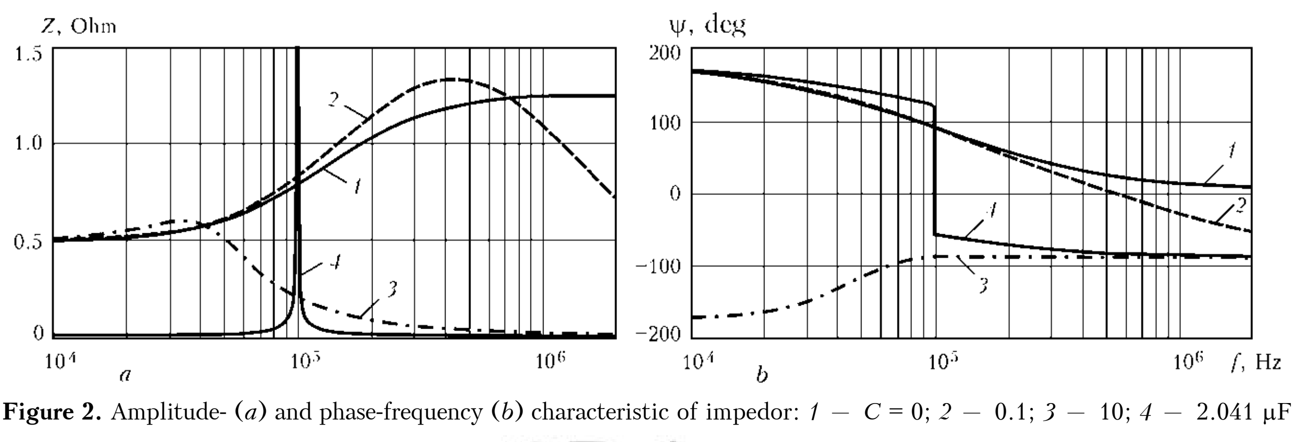

Figure 2. Amplitude- (a) and phase-frequency (b) characteristic of impedor: *1* — $C = 0$; *2* — 0.1; *3* — 10; *4* — 2.041 μF

In this case, the impedor is a low-frequency system, pass band of which represents itself frequency range from zero to cutoff frequency ω_C. It should be noted that the impedor frequency band is, roughly speaking, frequency range in which $Z(j\omega)$ value is close to 1. Accurate value of the cutoff frequency, certainly to significant extent, depends on figure ξ.

Differential equation of given circuit, made in relation to voltage $u(t)$ based on input circuit resistance, has the following appearance:

$$\theta R_{st0}C\,\frac{d^2u}{dt^2} + (R_{dif0}C + \theta)\,\frac{du}{dt} + u = \theta R_{st0}\,\frac{di}{dt} + R_{dif0}i. \qquad (2)$$

Varying of C value allows changing the coefficient at du/dt derivative. It is well known fact that sign and value of this coefficient determine nature of free oscillations in such dynamic system.

If $R_{dif0} < 0$ in equation (2), then regeneration, i.e. partial compensation of circuit loss, is possible due to feedback.

Let's determine the conditions of self-excitation scheme (see Figure 1, *a*) through study of characteristic equation of this system with internal feedback. It should be indicated that, if, for example $Y_{neg} = S_{dif} < 0$ (S_{dif} is the differential slope of VAC curve) is the negative active conductivity, introduced by the electric arc, then the condition of system self-excitation lies in compensation of the circuit loss.

It means that energy, dissipated in the circuit during period of own oscillations, in stationary mode, is exactly equal the energy, coming into the circuit from external sources per given period of time. Such a mechanism of self-excitation was termed as internal feedback [3, 5. 7]. *RLC*-circuit is an oscillating system in this case, and Y_{neg} (active element) being feedback element.

Roots γ_1 and γ_2 of characteristic equation (2) have real parts

$$\mathrm{Re}\gamma_{1,\,2} = -\frac{R_{dif0}C + \theta}{2\theta R_{st0}C}.$$

The system develops into unstable mode, when $\mathrm{Re}\gamma_{1,\,2}$ value becomes zero. At that since $(1 - R_{dif0}/R_{st0} > 0)$, then own continuous harmonic oscillations of the following form are received:

$$u_{own}(t) = A\sin(\omega_0 t + \varphi),$$

where A, φ are determined by initial values of $u(t_0)$ and $\dot{u}(t_0)$.

If condenser capacity C achieves the critical value $C_{cr} = -\theta S_{dif}$, then the characteristic equation is transformed into

$$d^2u/dt^2 + \omega_0^2 u = 0, \qquad (3)$$

where $\omega_0 = \omega_C\sqrt{1 - \dfrac{R_{dif0}}{R_{st0}}} = \dfrac{1}{\sqrt{LC}}\sqrt{1 - \dfrac{R_{dif0}}{R_{st0}}}$ is the frequency of own oscillations.

Critical value of negative resistance is found based on

$$R_{dif0\ cr} = -\theta/C.$$

It can be seen from the latter expression that the lower the condenser capacity C, the more the negative resistance, necessary for circuit self-excitation, is.

Figure 1, *a* shows the circuit with parameters $C = 1$ μF, $\theta = 10$ μs, $R_{dif0\,cr} = -10$ Ohm. If $R_{dif0} = -2$ Ohm, then $C_{cr} = 5$ μF.

It is obvious that electric arc with parallel capacity has stable arcing, until fulfillment of condition $\theta > -CR_{dif0}$.

Dissipative properties of the oscillating system can be characterized using coefficient of circuit attenuation α, relative damping $\xi = \alpha/\omega 0$ (dimensionless parameter) as well as coefficient of energy loss $\eta = 2\alpha/\omega_0$ in the system. Energy dissipation can be also estimated with the help of absorption coefficient ψ, related by simple approximated dependence with other characteristic of the process of energy dissipation in the system, namely logarithmic oscillation decrement δ: $\psi = 2\delta = 4\pi\alpha/\omega_0 = 4\pi \ln A_1/A_2$, where A_1, A_2 are the amplitudes of two neighbor oscillations relatively to set value. Approximated formula from work [7] $\eta = 2\delta/\sqrt{4\pi^2 + \delta^2} \approx \delta(1 - 0.0127\delta^2)/\pi$, error of which does not exceed 1 % at $\delta \le 3$, can be used at large δ values and at low damping $(\delta^2 << 6)\eta = \delta/\pi = \psi/2\pi$.

Circuit becomes unstable at $C > C_{cr}$. Introducing parameter

$$\alpha = \frac{1}{2}\left[-\theta/LC - R_{dif0}/L\right]\left(1 - \frac{R_{dif0}}{R_{st0}}\right) > 0,$$

differential equation is received:

$$\frac{d^2u}{dt^2} - 2\alpha\,\frac{du}{dt} + \omega_0^2 u = 0,$$

solution of which describes harmonic oscillations with amplitude exponentially increasing in time

$$u(t) = Ae^{\alpha t}\cos\sqrt{\omega_0^2 - \alpha^2}\,t + Be^{\alpha t}\sin\sqrt{\omega_0^2 - \alpha^2}\,t,$$

where A and B are the constants, depending on initial conditions.

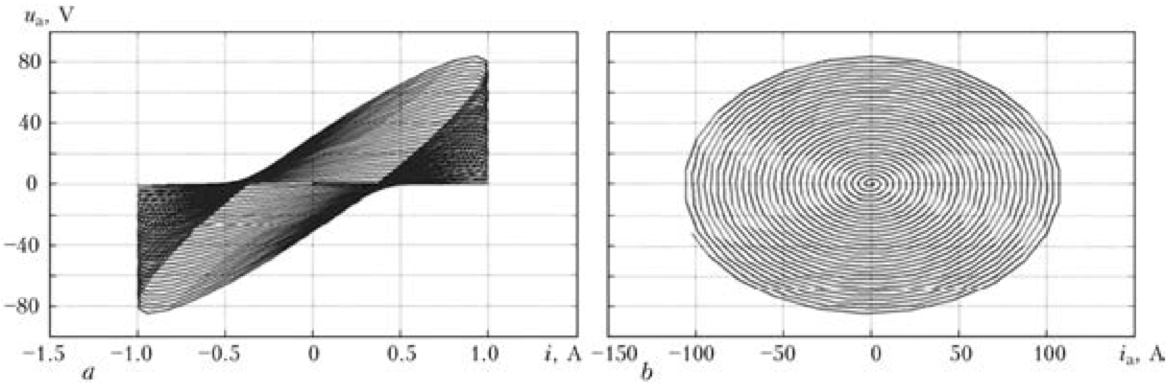

Figure 3. u–i-diagrams: a — $u_a = f(i)$; b — $u_a = f(i_a)$

If $\alpha << \omega_0$, then in accordance with (3) the basic frequency of auto-oscillations, appearing in linear mode, is close to the frequency of circuit own oscillations.

Referring to equivalent circuit (see Figure 1, b), it can be seen that current with complex amplitude I_m, coming from the current source, flow through resistance

$$Z_{eq}(j\omega) = Z(j\omega)R_i/[Z(j\omega) + R_i] =$$

$$= \frac{p\theta R_{st0} + R_{dif0}}{p^2\theta R_{st0}C + p[R_{dif0}C + \theta(1 + R_{st0}/R_i)] + R_{dif0}/R_i + 1}.$$

Simple transformations show that

$$Z_{eq}(j\xi) = \frac{R_{res.eq}}{1 + j\xi_{eq}}, \tag{4}$$

where $R_{res.eq} = R_{res}/(1 + R_{res})/R_i$ is the equivalent resistance of circuit at resonance considering discharge resistance R_i; $\xi_{eq} = \xi_0/(1 + R_{res})/R_i$ is the equivalent generalized detuning; ξ_0 is the dimensionless generalized detuning at $R_i = \infty$.

It can be assumed that effect of R_i lies in the fact that quality factor of the oscillating system reduces and becomes equal the equivalent quality factor

$$Q_{eq} = \frac{Q}{1 + R_{res}/R_i}.$$

According to the latter formula, reduction of effect of $-R_{res}$ on the oscillating system requires decreasing of resonance resistance R_{res} by means of R_i parallel connection.

Let's study parallel oscillating circuit of the following parameters: $\theta = 1$ μs, $R_{st0} = 1.25$ Ohm, $R_{dif0} = -0.49$ Ohm, $C = C_{cr} = 2.041$ μF, tuned for f_{res} frequency. Frequency of own oscillations in the circuit

$$\omega_0 = \frac{1}{\sqrt{\theta R_{st0}C}}\sqrt{1 - R_{dif0}/R_{st0} - \frac{C}{\theta}R_{dif0}^2/R_{st0}} =$$

$$= 0.626\cdot 10^6 \ (\mathrm{s}^{-1}); \quad f_0 = 99.7\cdot 10^3 \ \mathrm{Hz} \approx 100 \ \mathrm{kHz}.$$

Resonance frequency of oscillating system

$$R_{res} = \frac{R_{dif0}[(\theta\omega_0)^2R_{st0}/R_{dif0} + 1]}{(\theta\omega_0)^2R_{st0}/R_{dif0} + 1} = -10.66 \ (\mathrm{kOhm}).$$

Equivalent resistance of the circuit with resonance considering shunting effect R_i (R_i = = 10 kOhm) is

$$R_{res.eq} = 161.52 \ \mathrm{kOhm}.$$

$\xi_{eq} = 0$ in circuit resonating, therefore it follows from (4) that resonance coefficient of circuit transfer is

$$K_{res} = R_{res.eq}.$$

It is obvious that active resonance resistance of circuit R_{res} plays for this circuit a role of shunting resistor R without consideration of internal source and damping resistor (introduced by resistance R_i).

If $R_{res} < 0$ is the active constituent of input circuit resistance at resonance, and $1/R_i > 0$ is

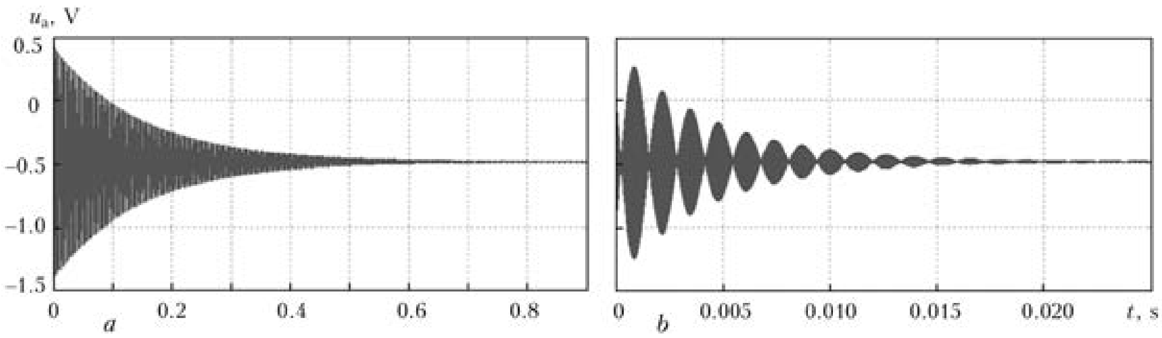

Figure 4. Circuit response on $R_i = 10$ (a) and 1 (b) kOhm switching function

the parallel conduction, introduced by current source and damping resistor, then the condition for providing of stability and absence of self-oscillation (stabilizing and damping) will take the form

$$-R_i/R_{\mathrm{res}} < 1.$$

If this condition is fulfilled, then damping, reflected by member with positive coefficient in the main equation of small oscillation, results in attenuation of oscillations. Studied system will be self excited, if negative resistance present in it is lower than introduced resistance $-R_{\mathrm{res}}/R_{i\ \mathrm{in}}$. It should be indicated that instability-promoted overvoltages influence the unit in whole.

Figure 3 shows an example of calculation results. At that the main conditions with $\theta = 1$ μs, $R_{\mathrm{dif0}} = -0.49$ Ohm, $C = 2.041$ μF were selected in such a way as to provide instabilities. It should be noted that appearing instability results in oscillations of voltage as well as arc current.

Spiral form of characteristic on *u*–*i*-diagram (see Figure 3, *b*) shows that voltage and arc current are phase-shifted relatively to each other.

It is useful to compare circuit transfer characteristics, obtained at different R_i/R_{res} values (Figure 4). If $R_i = 10$ Ohm, then the self-oscillations with negative attenuation are received (Figure 4, *a*).

At last, in general case, the system transfer characteristic (Figure 4, *b*) represents itself a quasi-harmonic attenuating oscillation with pulsations.

Effect of R_i depending on $R_i/|R_{\mathrm{res}}|$ is demonstrated in rate of reduction of current and arc voltage in time. If values of $R_i/|R_{\mathrm{res}}|$ relationship lie approximately between 0.95 to 0.5, it promotes significant attenuation of voltage (current) oscillations in comparison with $R_i = \infty$ case. If in contrast this relationship is more reduced, then the oscillations will continue attenuation, but relative change is already not so large. Moreover, $R_i/|R_{\mathrm{res}}|$ reduction promotes pulsation appearance.

Thus, resistance R_i for resistive damping should make approximately $R_i \approx |R_{\mathrm{res}}|$ in practical application.

Conclusions

1. Appearance of oscillation is possible in electric circuit with the electric arc. Set oscillation amplitude is determined by type of non-linear characteristic of the electric arc, included in the circuit.
2. Found frequency characteristics of input resistance of the circuit allow determining areas of system instability with complex or non-linear loading.
3. The oscillations attenuate with sufficiently low rate in the circuit, having only resistance damping at $-R_i/R_{\mathrm{res}} \approx 1$.
4. Capacity of condenser C can be such selected that the impedor would be damped in desired pass band.

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