COMPARATIVE ANALYSIS OF MODELS OF DYNAMIC WELDING ARC*

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There are number of dynamic arc models at present time. Therefore, the researchers when solving the specific tasks on investigation of processes in power source—welding arc system face with the problem of selection of a model providing complete description of the main arc peculiarities in each specific case. This paper is dedicated to objective estimation and comparison of models of dynamic arc and elaboration of recommendations for model selection on this basis.

Some progress in description of welding arc of constant length (non-consumable electrode) as part of electric circuit was achieved using mathematical model of dynamic arc, developed at the E.O. Paton Electric Welding Institute (PWI-MA) [1]. PWI-MA was further developed in works [2, 3]. Arc column in PWI-MA has been phenomenological considered as heat inertial macroobject, for which the following power balance is valid:

\[
\frac{dQ}{dt} = P - P_\theta,
\]

where \( Q \) is the internal energy of arc column; \( P \) and \( P_\theta \) is the input and output power, respectively. All isoenergetic identical states in PWI-MA are characterized by one parameter, i.e. arc state current \( i_\theta \) [1, 3]. It determines such characteristics of the arc column as static resistance

\[
R_{st}(i_\theta) = \frac{U_{col}(i_\theta)}{i_\theta},
\]

and output power

\[
P_\theta = U_{col}(i_\theta)i_\theta,
\]

where \( U_{col} \) is the function defining static volt-ampere characteristic (SVAC) of the arc column.

Input power is determined by arc column resistance as well as value of its transitional current:

\[
P = R_{st}(i_\theta)i_\theta^2 = \frac{U_{col}(i_\theta)}{i_\theta}i_\theta^2.
\]

Voltage on the arc column in dynamic is found from expression

\[
u_{col} = \frac{U_{col}(i_\theta)}{i_\theta}i,
\]

State current \( i_\theta \) for any representative point on plane, corresponding to arc column dynamic states in the coordinates of arc column voltage arc current \((u_{col}, i)\), is determined as x-coordinate of cross point of beam from the origin, passing through this point, with arc column SVAC. Work [3] shows that PWI-MA equation in differential form [2] corresponds to energy balance equation (1):

\[
\frac{dQ}{dt} = i^2 - i_\theta^2,
\]

which is electroengineering analog of equation (1). At that internal energy of arc column equals [3]

\[
Q = 2\theta \int_{0}^{i_\theta} U_{col}(i_{\theta})di_{\theta},
\]

where \( \theta \) is the time constant of arc column.

Since all other models use such a value as arc column conductivity \( g \) [4], PWI-MA equation...
is transferred in conductivity terms using formula (2), from which

\[ g = \frac{i_0}{U_{\text{col}}(i_0)}. \]  

(8)

PWI-MA equation is converted in \( g \)-form by differentiation of expression (8) by time and substituting found \( di_0/dt \) value in equation (6):

\[
\left[ \frac{20}{1 - G_{\text{dif}}(g)} \right] \frac{1}{g} \frac{dg}{dt} + 1 = \frac{i^2}{g^2U^2_{\text{col}}(g)} = \frac{i^2}{gP_0},
\]  

(9)

where \( G_{\text{dif}} = (dU_{\text{col}}/di_0)^{-1} \) is the differential conductivity. For making a comparison let us provide equations of arc column dynamic of all widespread models.

Cassie’s model [5]:

\[
\frac{\theta_C}{g} \frac{dg}{dt} + 1 = \left( \frac{i}{gU_C} \right)^2,
\]  

(10)

where \( \theta_C \) is the arc time constant in Cassie’s model; \( U_C \) is the static voltage on the arc column, which is constant in Cassie’s model.

Mayr’s model [6]:

\[
\frac{\theta_M}{g} \frac{dg}{dt} + 1 = \frac{i^2}{gP_M},
\]  

(11)

where \( \theta_M \) is the time constant in Mayr’s model; \( P_M \) is the output power, which is constant in Mayr’s model.

Zarudi’s model [7, 8]:

\[
\frac{\theta_Z}{g} \frac{dg}{dt} + 1 = \frac{i^2}{g^{k+1}X_k},
\]  

(12)

where \( \theta_Z = (1 + k)\theta \) is the value, which can be considered as time constant in Zarudi’s model; \( k \) is the index of plasma non-linearity depending on gas nature, in which arc burns; \( X_k \) is the constant at fixed \( k \).

Schellhase’s model [9]:

\[
\frac{\theta_S}{g} \frac{dg}{dt} + 1 = \frac{1}{gU_{\text{col}}(i)}
\]  

(13)

where \( \theta_S \) is the time constant in Schellhase’s model.

Comparison of the models shows that the time constants are different in all models. Let us consider what are the conditions when expression

\[ \tau = \frac{20}{1 - g/G_{\text{dif}}(g)} \]  

(14)
in equation (9) is the constant.

Obviously, for this it is necessary that

\[ \frac{g}{G_{\text{dif}}(g)} = \frac{i_0}{U_{\text{col}}(i_0)} \frac{dU_{\text{col}}}{dt} = n = \text{const}. \]

Separating the variables and integrating the latter equation the following is received:

\[
U_{\text{col}}(i_0) = U_0 \frac{i_0^n}{\left( \frac{i_0}{i_0} \right)^n},
\]  

(15)

where \( U_0 \) and \( I_0 \) are the coordinates of one of SVAC fixed points.

Thus, Cassie’s, Mayr’s and Zarudi’s models follow from PWI-MA (9) in a special case, when SVAC has power form with index of power \( n \), and equation (9) is written as

\[
\frac{20}{1 - n} \frac{dg}{dt} + 1 = \frac{U_0^n}{i_0^n} \frac{i^2}{g^{1-n}}.
\]

Cassie’s model (10) is received at \( n = 0 \). Then \( U_C = U_0 \) and \( \theta_C = 20 \). Mayr’s model (11) takes place at \( n = -1 \). In this case \( P_M = U_0I_0 \) and \( \theta_M = \theta \). Zarudi’s model (12) is realized at \( n = -(1 - k)/(1 + k) \). Then

\[
X_k = I_0^2 \left( \frac{U_0}{I_0} \right)^{1+k} \quad \text{and} \quad \theta_Z = (1 + k)\theta = \frac{20}{1 - n}.
\]

Schellhase’s model (13) can not be received from PWI-MA (9) under any conditions. It can be superposed neither with Cassie’s models no with Zarudi’s model. Obviously, it is explained by the fact that Schellhase’s model [9] was built on concepts, which did not use power balance equation (1).

Specific attention is to be given to interpretation of coefficient \( \tau \) from formula (14). It was mentioned above that this coefficient remains constant in the case of SVAC power function (15). Therefore, it is called an arc time constant in different models. In general case \( \tau \) value depends on conductivity and can not be interpreted as arc time constant. Moreover, as it is shown in work [3], the transfer processes in the circuit with arc generally can not be characterized by process time constant. This conclusion matches with the results of experimental investigations in works [10, 11]. Thus, PWI-MA eliminates existing contradiction on variability of arc time constant. The arc time constant \( \theta \) in PWI-MA is the invariable value, but it is not time constant of the process. And generally coefficient \( \tau \) is the variable value.
It follows from mentioned above that PWI-MA applicable to welding arc has series of advantages in comparison with other methods. PWI-MA is the most general of all models. It can use SVAC of any type (including areas with independent and growing SVAC), that is very important in welding arc modelling. PWI-MA, in terms of electric engineering, uses a value of state current, that has apparent advantages in comparison with other models, which use values of conductivity or arc column resistance. PWI-MA allows easily determining such important energy parameters of the arc column as output power and internal energy of the arc column. Formula (7) for internal energy of the arc column, obtained with the help of PWI-MA, allows expanding sphere of application of dynamic arc equations, including for the cases of arcs with varying in time arc length \( l_a(t) \) as well as moving and blown arcs, that is in principle inaccessible for alternative arc models.

Using equality

\[
U_{\text{col}}(i_0) = l_a(t)E_{\text{col}}(i_0),
\]

where \( E_{\text{col}}(i_0) \) is the average value of electric field intensity in the arc column (SVAC of the arc is determined particularly for this value of intensity, in the case of cylindrical arc, \( E_{\text{col}}(i_0) \) agrees with real electric field intensity in the arc column). Let us transfer equation (7) in the following form:

\[
Q = 20l_a(t) \int \limits_0^{i_0} E_{\text{col}}(i_0)di_0.
\]

Here out

\[
\frac{dQ}{dt} = \frac{\partial Q}{\partial i_0} \frac{di_0}{dt} + \frac{\partial Q}{\partial l_a} \frac{dl_a}{dt} = \frac{dQ}{dt} = 20l_aE_{\text{col}}(i_0) \frac{dt_a}{dt} + 20 \frac{dl_a}{dt} \int \limits_0^{i_0} E_{\text{col}}(i_0)di_0.
\]

Substituting this expression together with (3) and (4) in equation (1), PWI-MA equation for changing arc length is received in general case:

\[
0 \frac{di_0^2}{dt} = i^2 - i_0^2 \left[ 1 + \frac{\nu_l}{l_a} \int \limits_0^{i_0} E_{\text{col}}(i_0)di_0 \right],
\]

where \( \nu_l = dl_a/ dt \) is the rate of arc length change.

The latter equation assumes the following form of record for general case

\[
0 \frac{di_0^2}{dt} = i^2 - i_0^2 \left[ 1 + \frac{\nu_l}{l_a} \right].
\]

and for particular case of power approximation of SVAC curve (15)

\[
0 \frac{di_0^2}{dt} = i^2 - i_0^2 \left[ 1 + \frac{2\nu_l}{(1 + n)l_a} \right].
\]
Equation (25) is expedient to be used for description of processes with plasma arc application, when longitudinal gas blow is inseparable part of the process. Equation (26) shall be used in transverse blowing as well as for different cases of arc movement, including arc movement in magnetic field, at that $v_{bl}$ is considered as arc movement rate. It should be noted that different combinations of equations (19), (24) and (25) are possible. For example PWI-MA equation

$$0 \frac{dt_0}{dt} = r^2 - i_0 \left(1 + \frac{|v_{bl}| + v}{l_a} Q \right)$$

(27)

is valid in arc length change and longitudinal arc blow.

Conclusions

1. Mathematical model of dynamic arc is valid for any types of SVAC and allows solving the problems in the electroengineering terms. It can be used in the cases of varying length arcs (including the consumable electrode arc), moving arcs and gas blown arcs, and provides the method for determination of internal energy and output power, that is impossible in alternative models.

2. Obtained results are valid not only for welding, but for plasma arcs and rupturing arcs in electric apparatuses.

3. Using the computer PWI-MA equations allows investigating evolutionary processes in power source–arc system for various cases of gas-shielded welding, namely consumable-electrode in inert gases and mixture of active gases and tungsten-electrode in inert gases welding.


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The monograph is devoted to presenting the results of research of fundamental properties of electric arc as a nonlinear part of electrical circuits. The revealed regularities and mechanisms of deterministic chaos in these circuits as well as scenarios of its development have been described. A special attention is paid to the original mathematical methods for investigation of nonlinear dynamical systems. All results have been illustrated.

The monograph is intended for a wide range of professionals in the fields of theoretical electrical engineering and nonlinear dynamic systems. It could be useful to scientists, students and postgraduate students.

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