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NUMERICAL MODELING OF STRESS-STRAIN STATE OF ELEMENTS MANUFACTURED BY 3D PRINTING

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Manufacture of parts by the method of 3D printing, in particular, applying FDM (Fusing Deposition Modeling) technology, is a promising trend in many branches of mechanical engineering, architecture, construction, medicine, etc. This range of problems challenges three main directions of studies: FDM 3D printing technology, materials science and mathematical modeling of processes for evaluation of functional qualities, in particular, strength of products. This work is devoted to the third direction: evaluation of stress-strain state of products manufactured by 3D printing using FDM technology. The paper considers three stages of solving this problem: 1 — mathematical formulation of the problem, which includes universal balance relations, determining equations of mechanical behaviour of the material; 2 — method of numerical solution of the problem; 3 — solving specific problems in order to determine patterns of thermomechanical processes and provide recommendations for technological settings of 3D printing. 10 Ref., 12 Figures.

Keywords: additive technologies, FDM 3D printing, mathematical modeling, stress-strain state

Modeling by the method of layer-by-layer surfacing or FDM 3D printing with polymer materials due to its versatility, simplicity, multifunctional capabilities and affordability is considered to be the most common 3D printing technology in the world, based on which millions of 3D printers — from the cheapest to industrial three-dimensional printing systems are operating [1, 2]. To create products by FDM 3D printing, a polymer material is used in the form of a thread (filament) from different thermoplastic materials that are supplied in coils. There may be two standard diameters of a filament: 1.75 and 3.0 mm depending on the printer specification [3].

As in all 3D printing technologies, the first step on the way of manufacturing a physical object is building its digital 3D model [4] in special programs (Autodesk 3DsMAX, ZBrush, Maya, Blender, SolidWorks, etc.), which in the STL format is transmitted to the software of a 3D printer. Before launching the printing process, the necessary 3D printing settings (speed, temperature, etc.) are selected and the model in the slicer-program is automatically divided into horizontal layers to calculate the ways for movement of the extruder (printing head) — a device equipped with a mechanical drive for feeding a filament, a heating element for its fusion and a die, through which the extrusion is directly carried out — pushing out of the fused polymer material to the surface of a product (Figure 1, *a*).

At the same stage, if necessary, supporting structures are generated if overhanging elements are pres-

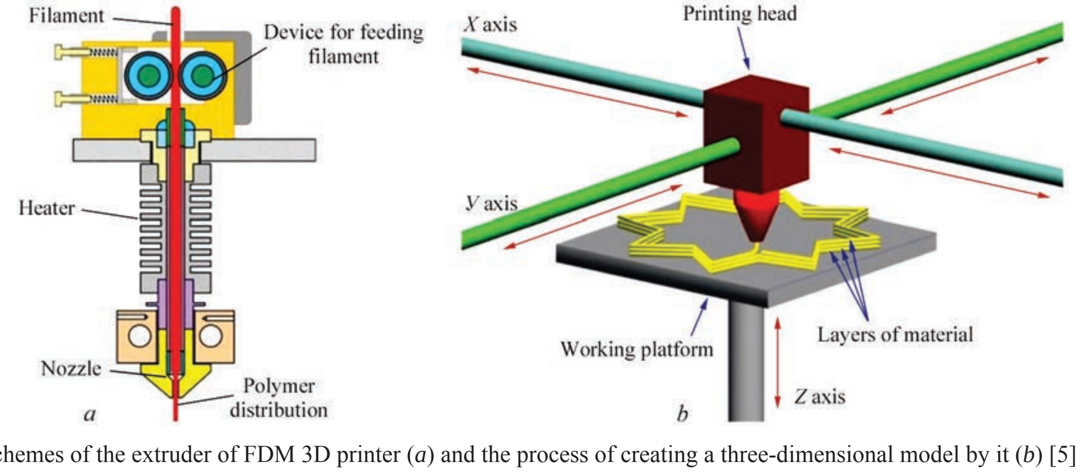

Figure 1. Schemes of the extruder of FDM 3D printer (*a*) and the process of creating a three-dimensional model by it (*b*) [5]

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ent in the model. When the preparatory part is completed, the control code (G-Code) is generated for the 3D printer based on digital data and selected settings. Further, a filament from the coil is unwound to be introduced into the extruder and a process of 3D printing itself is launched: the extruder fuses a filament and feeds a polymer material with a high precision by thin layers to a working surface of the 3D printer according to the printing algorithm and a digital 3D model.

After deposition of a layer, a polymer material is cooled and getting solid, and the platform, on which an object is formed, lowers to the level equal to the thickness of the deposited layer. The movement in three planes of the head and platform (Figure 1, *b*) is set by the algorithm developed in advance with the help of special software. When the process of building a product is completed, auxiliary structures are removed (manually or dissolved in a special solution), and a finished product can be used in a printed form or exposed to any method of further treatment.

For evaluation of the strength of parts, information on current and residual stresses, as well as distortion, especially for thin-walled elements plays an important role. Therefore, the development of methods for mathematical modeling of processes and evaluation of the mentioned settings is an actual issue in this range of problems.

Problem formulation. In this work, a simplified thermoelastic formulation of the problem is used. This model does not take into account the relaxation effect and provides the upper evaluation of current and residual stresses [6]. A more precise formulation that takes into account viscoelastic properties of the material, structural transformations (crystallization) in the material, as well as their influence on physical and mechanical properties of the material, will be considered in the next publications.

Balance equations. Element building is considered in the rectangular coordinate system. Material is considered to be isotropic. The initial formulation of the problem in an invariant form includes kinematic relations,

equation of thermal conductivity

$$c_v \dot{\theta} = \operatorname{div}(k \operatorname{grad} \theta) + Q, \tag{1.1}$$

of quasi-static equilibrium

$$\operatorname{div} \underset{\sim}{\sigma} = 0, \tag{1.2}$$

boundary and initial conditions $\theta = \theta_0$ at $t = 0$; $-k\vec{n} \cdot \operatorname{grad}\theta = -q + \gamma(\theta - \theta C)$

$$\underset{\sim}{\sigma} \vec{n} = 0, \tag{1.3}$$

where θ is the temperature; σ is the tensor of stresses; *Q* is the power of a volumetric heat source; *q* is the set heat flow; c_v and k are the coefficients of heat capacity and heat conductivity; γ is the coefficient of heat transfer; θ_m is the temperature of surrounding medium; θ_0 is the initial temperature; $\vec{n}$ is the external normal to the body surface; $\underset{\sim}{\sigma}$ = , *i*, *j* = *x*, *y*, *z*; *i*, *j* = *x*, *y*, *z*.

Further, these equations will be modified taking into account the process of building.

For 2D state strained in the plane *Oxy*, we have

$$\sigma_{zz} = \sigma_{zx} = \sigma_{yz} = 0,\ u_i = u_i(x, y),\ \varepsilon_{ij} = \varepsilon_{ij}(x, y),$$
$$\sigma_{ij} = \sigma_{ij}(x, y),\ \theta = \theta(x, y).$$

Thermomechanical behaviour of the material is described with the help of the following relations

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^e + \underset{\sim}{\varepsilon}^\theta; \quad \underset{\sim}{\varepsilon}^\theta = \alpha(\theta - \theta_0)\underset{\sim}{I}; \tag{1.4}$$

$$\underset{\sim}{s} = 2G\underset{\sim}{e}, \quad tr\underset{\sim}{\sigma} = 3K_v tr(\underset{\sim}{\varepsilon} - \underset{\sim}{\varepsilon}^\theta), \tag{1.5}$$

here $\underset{\sim}{\varepsilon}^e$ and $\underset{\sim}{\varepsilon}^\theta$ are elastic and thermal components of deformation; *s* and $\underset{\sim}{e}$ are deviators of stress tensors and deformations; *G* and K_v are shear and bulk modulus; tr is the trace of tensor; I is a single tensor.

Model of building bodies. Let us consider the modification of relations (1.4), (1.5), taking into account the process of building [7–9]. Let us assume that the problem is solved by the finite elements method. Let the building process is controllable, i.e., the rate of building and final body configuration are known. The simplest variant of the solution algorithm is as follows. A building body configuration is covered by the fixed CE-mesh. CE-mesh covers both the body to be built in the original configuration, as well as all layers to be built in the future. Therefore, the mesh (number of nodes) does not change in the process of numerical modeling. Another approach consists in the fact that the mesh is increased as a result of attachment of building elements. In the area occupied by a source body, the properties are determined by the body material. Initially, to the elements being built, the properties of the «cavity» material are attributed, which is considered to be thermoelastic with the following characteristics

$$E \approx 0,\ \upsilon \approx 0,\ \alpha = \alpha_f,$$

where *E* is the Young's modulus; υ is the Poisson's ratio; α_f is the coefficient of linear thermal expansion of a building material. The thermophysical properties of the «cavity» are accepted the same as in a building material. Therefore, the element is «empty» only in terms of mechanics. In the process of filling, which is considered as a process developing in time, «empty» elements of the CE-mesh will be filled with a building material. It is important to keep in mind that in the process of filling the elements (building), the whole CE-mesh is deformed, which covers both the source body as well as «empty» elements adjacent to the body.

Let at the time of filling t^*, some empty element $\Delta V(t^*)$ of the mesh has a deformation ε^*_{ij} and let it be filled with a material having a temperature θ^*. It is assumed that the material of the building elements up to the contact with the surface of the body is unstrained:

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{xz} = 0 \text{ at } t = t^*. \quad (1.6)$$

In the frames of this model of building, there is a filling of an element having a preliminary deformation ε^*_{ij}, a building material with a temperature θ^*. Thus, the conditions (1.6) mean that

$$\sigma_{ij}(\varepsilon^*_{kl}, \theta^*) = 0 \text{ in } \Delta V(t^*). \quad (1.7)$$

In order to that the determining equations of a building material (1.5) meet the condition (1.7), it is necessary and enough to modify the equation (1.5) as follows:

$$\begin{aligned} &\underset{\sim}{s} = 2G_f(\underset{\sim}{e} - \underset{\sim}{e}^*), \\ &tr\underset{\sim}{\sigma} = 3K_f tr(\underset{\sim}{\varepsilon} - \underset{\sim}{\varepsilon}^* - \alpha_f(\theta - \theta^*)\underset{\sim}{I}). \end{aligned} \quad (1.8)$$

Here the lower index f shows that the settings relate to the material of the building volume. Therefore, in order to meet the condition of building (1.6), all the elements being built should have the determining equations, $\underset{\sim}{\varepsilon}^*$ individualized by those specific values of deformation and temperature θ^*, at which their filling occurred. Therefore, the state (ε^*_{ij}, θ^*) for these elements can be interpreted as «own», as far as it does not cause stresses.

Problem formulation for building bodies. Taking into account the results mentioned in the preceding paragraph, the mathematical problem includes the following relations:

- equations of equilibrium (1.2) and thermal conductivity of (1.1)
- determining equations for base material

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^\theta_{ij}; \quad (1.9)$$

$$s_{ij} = 2Ge_{ij}, \quad \sigma_{kk} = 3K_V(\varepsilon_{kk} - \varepsilon^\theta_{kk}); \quad (1.10)$$

- determining equations for material being built in a component form

$$\begin{aligned} &s_{ij} = 2G\,(e_{ij} - e^*_{ij}), \\ &\sigma_{kk} = 3K_f(\varepsilon_{kk} - \varepsilon^{\theta^*}_{kk} - \varepsilon^*_{kk}), \end{aligned} \quad (1.11)$$

where G and K are the shear and bulk moduluses; Q is the heat source; λ and c_v are the coefficients of thermal conductivity and bulk heat capacity

$$\varepsilon^\theta_{ij} = \alpha(\theta - \theta_r)\delta_{ij}; \quad \varepsilon^{\theta^*}_{ij} = \alpha(\theta - \theta^*)\delta_{ij}, \quad (1.12)$$

here θ is the current temperature; θ_r is some reference temperature; α is the coefficient of linear thermal expansion.

Numerical method of problem solution. *Variational formulation of the problem.* The three-dimensional problem of the thermomechanical state of building parts is solved numerically using the finite element method [9]. Lagrange variational formulation of the problem in the Cartesian system of $Oxyz$ coordinates has the following form

$$\begin{aligned} \delta I = &\int_F \left[\lambda\left(\frac{\partial\theta}{\partial x}\delta\left(\frac{\partial\theta}{\partial x}\right) + \frac{\partial\theta}{\partial y}\delta\left(\frac{\partial\theta}{\partial y}\right) + \frac{\partial\theta}{\partial z}\delta\left(\frac{\partial\theta}{\partial z}\right)\right) + \right. \\ &\left. + \left(c_v\dot{\theta} - Q\right)\delta\theta\right] dxdydz + \\ &+ \int_S (-q + \gamma(\theta - \theta_c))\delta\theta ds = 0; \end{aligned} \quad (2.1)$$

$$\begin{aligned} \delta\Phi = &\int_F \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{zz}\delta\varepsilon_{zz} + 2\sigma_{xy}\varepsilon_{xy} + \right. \\ &\left. + 2\sigma_{yz}\varepsilon_{yz} + 2\sigma_{zx}\varepsilon_{zx}\right) - \\ &- \int_S \left(t_{nx}\delta u_x + t_{ny}\delta u_y + t_{nz}\delta u_z\right) ds = 0, \end{aligned} \quad (2.2)$$

wherein δI is the variation of the functional for the problem of thermal conductivity; $\delta\Phi$ is the variation of the functional for the problem of mechanical equilibrium; t_{nr} and t_{nz} are the components of stress vector on the contour; u_r, u_z are the radial and axial components of the movement vector; V and S are the volume and surface of the body.

The equation of thermal conductivity is integrated by time using an implicit scheme. In this case, the linearization of the problem is achieved due to the fact that the characteristics dependent on the temperature are calculated for the previous time step. The distribution of temperature calculated for the time is used to solve the problem of mechanics. From the condition of the stationarity of the functional $\delta\Phi = 0$ (2.2), we obtain the system of algebraic equations for the movements in the nodal points. At the same time, the temperature included in the functional is taken by a constant element and does not vary. Based on the found nodal displacements, the deformations and stresses are calculated at the integration points, which are then averaged by a finite element method.

From the condition of the stationary functional $\delta I = 0$ to determine the nodal values of the temperature θ_i, we obtain a system of linear differential equations of the first order in time. The accuracy of the calculation depends on the number of finite elements. The required density of mesh of elements is determined by comparing the solutions of the problem at a different number of finite elements.

Object of investigation. The configuration of a building element is shown in Figure 2.

The sizes of the element: a = 6 mm, b = 2 mm, h = 0–50 mm, Δh = 0.14–0.42 mm.

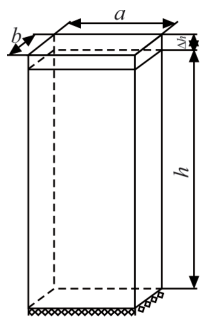

Figure 2. Building element

Let us assume that one layer $\Delta h = 0.14$ mm is built for 1.1 s (from the calculation: the rate of building of element 0.14×0.14 mm is 80 mm/s, i.e. the building time for one layer is 0.14 mm, at the end it is 1.1 s). Next we will consider three cases of building: by layers of 0.14 mm for 1.1 s per a layer, 360 layers; 0.28 mm for 2.2 s per a layer, 180 layers; 0.42 mm for 3.2 s per a layer, 120 layers, i.e. the rate of building (increase in height) in these three cases is the same.

On the lower surface the following conditions of fixing are set: $u_y = 0$, $\sigma_{xy} = 0$, $y = 0$.

Thermomechanical properties of the material. The material of the object is a PLA polymer with temperature-dependent properties.

The dependence of the specific heat capacity on temperature is shown in Figure 3.

The dependence of the Young's modulus of elasticity on temperature, taken from [10], is shown in Figure 4. Other settings are $\rho = 1210$ kg/m^3 is the density, $\nu = 0.35$ is the Poisson's ratio, $\alpha = 41\cdot10^{-6}$ K^{-1} is the coefficient of thermal expansion, $\lambda = 0.13$ W/m^2·K are the coefficients of thermal conductivity. On free surfaces a heat exchange with an ambient temperature of $\theta_m = 20$ °C with a heat transfer coefficient $\gamma = 30$ W/m^2·K occurs.

During building, the base temperature is maintained at 50 °C. Building occurs by fusion the material at 200 °C.

Results of calculations. Figure 5 shows the division of the area on finite elements at $\Delta h = 0.28$ mm. Here the resulting division is shown when all the layers are already built.

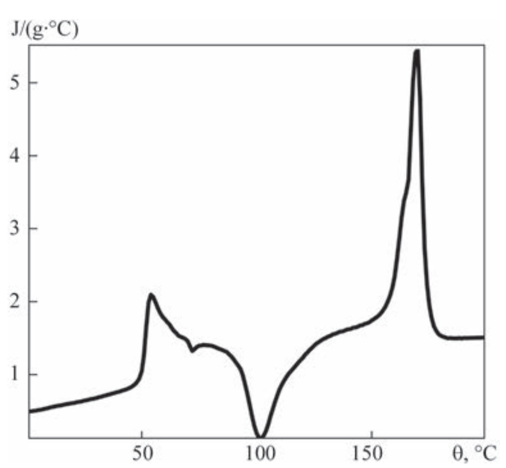

Figure 3. Dependence of specific heat capacity on temperature

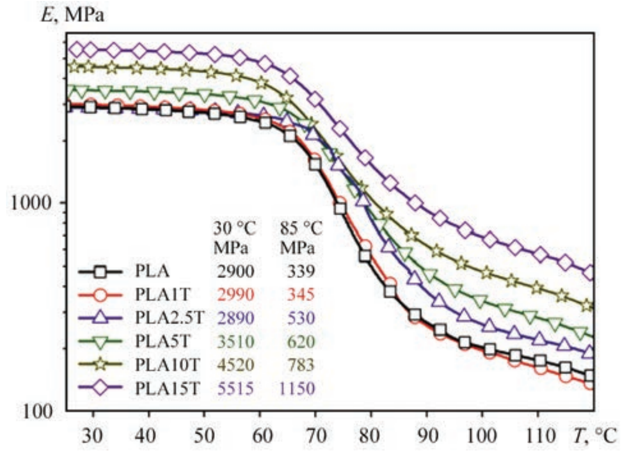

Figure 4. Dependence of the Young modulus of elasticity E on temperature

Figure 6 shows the isolines of the temperature distribution at different moments of building (Figure 6, *a*, *b*) and at the moment of cooling (Figure 6, *c*).

Figure 7 shows the variation in temperature over time at the points 1, 2 and 3, shown in Figure 5. Points 1 and 2 appear at the moment when the 90th layer is built. The point 3 corresponds to the highest point of the element at different moments of time.

Figure 8 shows similar curves for the stress σ_{yy} at the points 1 and 2.

Let us note that residual stresses in the thermoelastic problem are formed as a result of applying a heated layer on the already cooled previously deposited layer. The classic problem is mounting a heated bushing on the shaft (hot press fit), when residual compressive stresses can occur without plastic deformations.

It is seen that the residual stress distribution is formed at the temperature change from 80 °C to 50 °C. This is associated with the temperature depen-

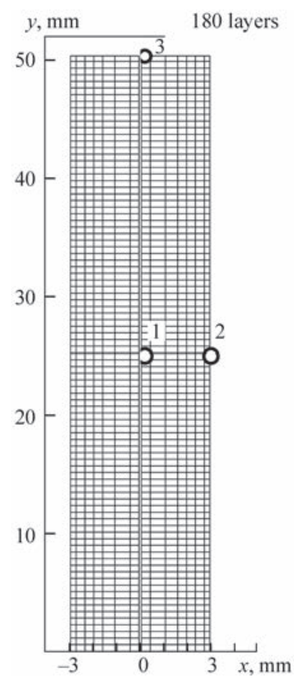

Figure 5. Division of area on finite elements

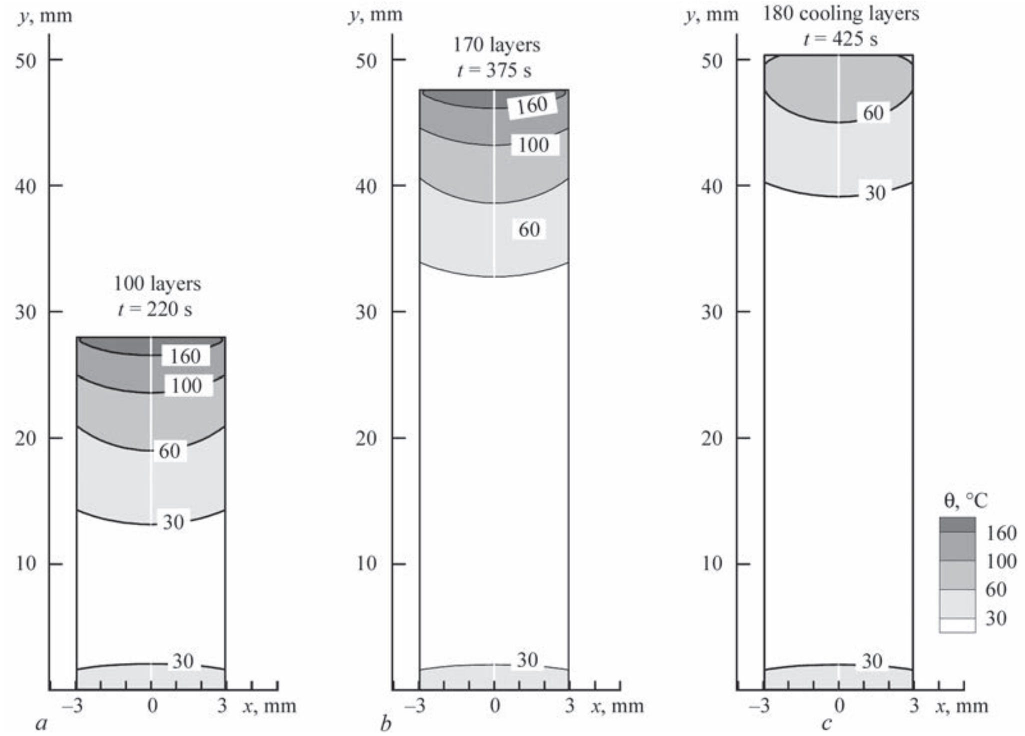

Figure 6. Isolines of temperature distribution θ at different moments of building

dence of the Young's modulus (see Figure 4). In this temperature range an increase in the rigidity of the material is observed.

Figure 9 shows the isolines of the residual distribution of σ_{yy} in the *Oxy* plane.

Let us study the influence of the thickness of building layer of the material on residual stresses. Let us consider three cases.

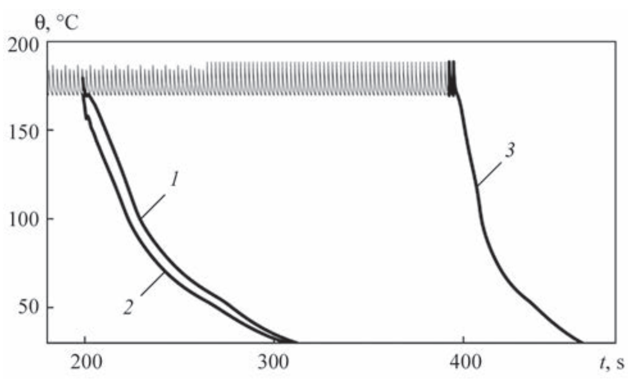

Figure 7. Temperature change over time at the points 1, 2 and 3

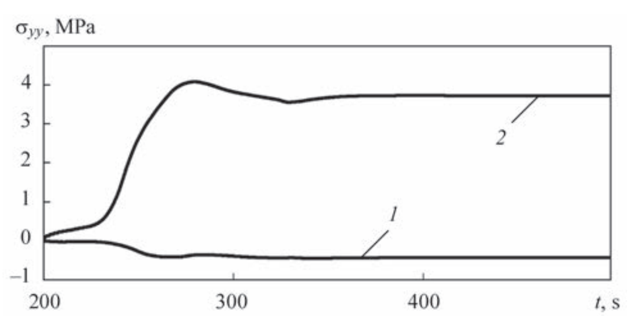

Figure 8. Change in σ_{yy} stress over time at the points 1 and 2

1. Building with layers of 0.14 mm. The results of the calculation are shown in Figure 10. Figure 10, *b* shows changes of σ_{yy} in the scale of the layers.

2. Building with layers of 0.28 mm. The results of the calculation are shown in Figure 11.

3. Building with layers of 0.42 mm. The results of the calculation are shown in Figure 12.

It is seen that the level of σ_{yy} stresses is decreased with an increase in thickness of a building layer (Figures 10, *a*; 11, *a*; 12, *a*). The smaller the thickness of building, the higher the average stress level, whereas

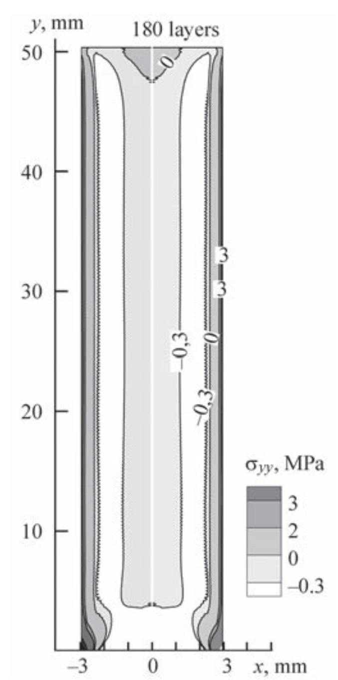

Figure 9. Residual distribution of longitudinal stress

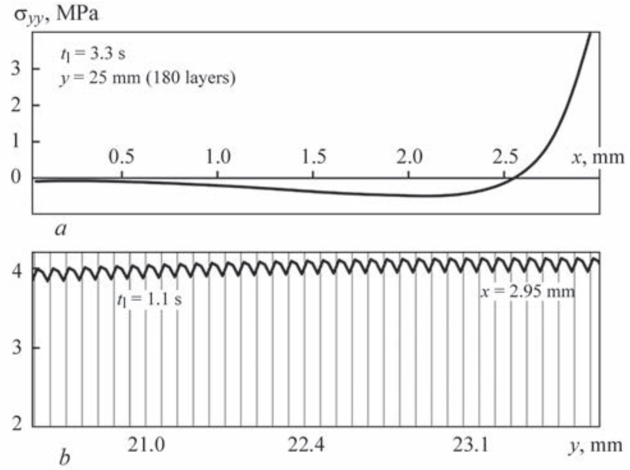

Figure 10. Building by layers of 0.14 mm, 360 layers: *a* — σ_{yy} in the cross-section $y = 0.02$ m; *b* — in the cross-section $x = 0$, along the axis *y*

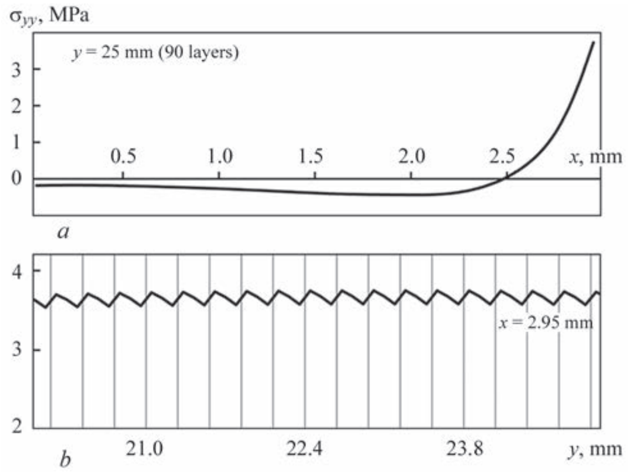

Figure 11. Building by layers of 0.28 mm, $t = 2.2$ s, 180 layers: *a* — σ_{yy} in the cross-section $y = 0.02$ m; *b* — in the cross-section $x = 0$, along the axis *y*

the variations within the boundaries of the built layer differ insignificantly (Figures 10, *b*; 11, *b*; 12, *b*).

Conclusions

1. In this work, a thermoelastic model of the stress-strain state of elements, manufactured by the method of additive molding by FDM 3D printing technology was developed. Namely, this is a model of multilayer building of bodies based on the theory of building bodies. Moreover, a finite element method by calculation of the thermoelastic state of layered objects was proposed.

2. The results of carried out investigations and calculations of the current and residual thermomechanical state of a particular building plate element at different moments of building and at the moment of cooling indicate that the residual stress distribution is formed by changing the temperature from 80 to 50 °C, which is associated with the temperature dependence of the

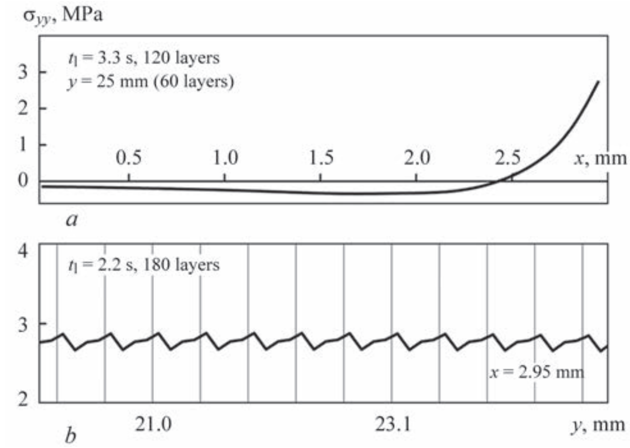

Figure 12. Building by layers of 0.42 mm, $t = 3.3$ s: *a* — σ_{yy} in the cross-section $y = 0.02$ m; *b* — in the cross-section $x = 0$, along the axis *y*

Young's modulus of the test material and an increase in its rigidity in the specified temperature range.

3. Evaluation of the influence of the thickness of a building layer of the material (0.14; 0.28; 0.42 mm) on the residual stress state of the elements showed that the level of σ_{yy} stresses decreases with an increase in the thickness of a building layer.

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