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MATHEMATICAL MODEL OF DETERMINATION OF RESIDUAL STRESSES AND STRAINS IN FRICTION STIR WELDING OF ALUMINIUM ALLOY

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ABSTRACT

A rater simple and efficient mathematical model of the process of friction stir welding (FSW) was developed, which is focused on fast determination of residual welding stresses and strains with engineering precision. The model is based on application of the method of thermoelastoplastic deformation of the material, which is used at modeling of arc welding, but instead of the model of arc heat source a model of heat evolution from the working tool friction against the material of the joint element was developed. The model also takes into account the conditions specific to FSW of rigid restraint of the joint elements during welding. The developed FSW model was used to conduct calculations of a butt joint of plates from AMg6 aluminium alloy and to present the results of the characteristic distribution of residual stresses and plastic strains, compared to arc welding of the butt joint. Calculation results derived using the developed model confirm the conclusions of other researchers that at FSW of aluminium alloys undesirable residual stresses and strains also form, but they are lower than with the traditional arc welding methods. Developed model can be effectively used for on-line calculation definition of residual stresses and plastic deformations in the zone of welded joints produced by FSW, with the purpose of further assessment of welded joint strength or prediction of general deformations of large-sized structures. Ways of further improvement of the developed model were outlined with the purpose of further increase of prediction accuracy, also by allowing for degradation of mechanical properties (softening) of the aluminium alloy during heating.

KEYWORDS: friction stir welding, aluminium alloy, residual stresses, plastic strains, mathematical modeling

INTRODUCTION

Nowadays the technology of friction stir welding (FSW) is becoming ever wider accepted when making butt joints of critical structures in such industries as aerospace, shipbuilding, automotive and railway transport, etc. Considering the high requirements to structures operating at high loads, determination of residual stresses and strains in FSW welded joints and development of the mathematical model of their determination is an urgent task.

During FSW, when the working tool, immersed into the welded joint metal, rotates and advances along the weld line, an intensive process of the tool shoulder friction against the surface of the joint elements and of the surface of the pin against the material occurs in the product volume. The process of metal stirring in the volume near the surface of contact of the pin and shoulder with the joint elements also takes place The tool shape and FSW process parameters can significantly influence the welded joint quality. [1, 2].

For effective modeling of this process, different researchers use diverse approaches, depending on the posed objectives. This can be prediction of the quality of welded joint formation for optimization of the welding technology or prediction of residual welding stresses and strains, which influence the performance of such a structure.

The simplest mathematical models of the heat source at FSW [3] define heat evolution only as a result of the process of the tool friction against the material of the joint elements. More detailed models further take into account the contribution of plastic strains into element heating. In such models [4, 5], in addition to the friction process, it is also necessary to predict the process of material stirring, so as to determine the balance of contributions of friction and plastic strains into the joint element heating.

In work [6] not a constant, but variable temperature-dependent coefficient is used in FSW model. In the mathematical model, presented in [6], also the amount of plastic work, converted into heat, is equal to 80 %, although the authors note that in many studies this value does not reach even 5 %.

In works [3, 5–9] the finite element model, alongside the welded joint, also includes the working tool and fixture for fastening its elements that allows determination of heat losses of the heating source into the tool and the fixture. Other researchers [4] use semi-analytical thermal model of FSW process which reduces the computation time.

All the models of the stress-strain state determination at FSW take into account the action of thermal stresses on the processes of elastoplastic deformation of the material during welding and further cooling [9]. The main feature of modeling the FSW process

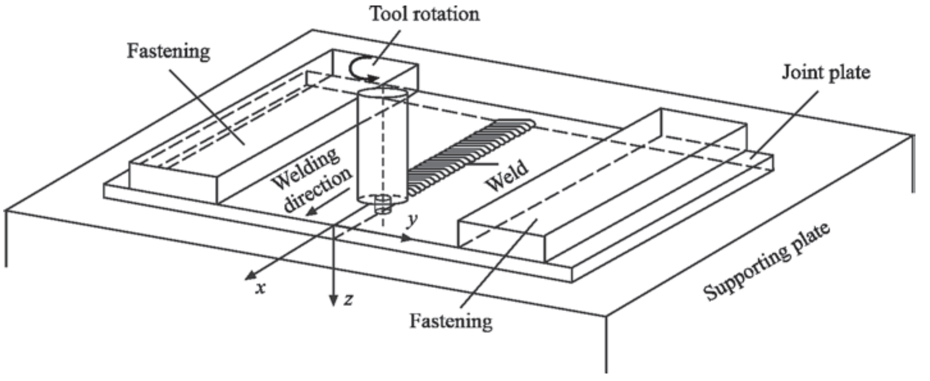

Figure 1. Scheme of FSW process

is rather rigid fastening of the joint elements during welding [6, 8].

MATHEMATICAL MODEL

Analysis of the available works on mathematical modeling of the processes of thermoelastoplastic deformation and mass transfer at FSW resulted in development of a rather simple model, aimed at quick determination of residual welding stresses and strains with engineering precision with the purpose of further assessment of welded joint strength, under the conditions of operational loading or prediction of total strains of large-sized structures with a large number of welded joints by shrinkage function method [10].

It is known that heat evolution at FSW, produced as a result of deformation, does not exceed 5 % of the total amount of heat evolution [11]. Thus, in the proposed model heat evolution from metal deformation can be neglected, for quick derivation of the data on residual welding stresses and strains at FSW of aluminium alloy elements. More over, in order to simplify the model, while preserving engineering precision of prediction, the following factors were ignored: dependence of the friction coefficient on material temperature, heat removal into the working tool and fixture for fastening the joint elements (supporting plate and clamps), partial lowering of heat evolution from friction as a result of material stirring, as well as the softening effect — lowering of mechanical properties of the aluminium alloy during heating [12].

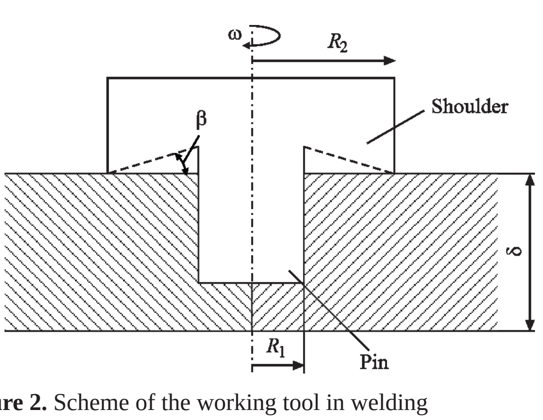

Figure 2. Scheme of the working tool in welding

Modeling of temperature fields at FSW was performed using the equation of nonstationary heat conductivity, which takes into account the bulk welding heat source $W(x, y, z, t)$

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+ +W(x,y,z,t)=c\rho\frac{\partial T}{\partial t}, \quad (1)$$

where ρ is the material density; c is the specific heat content, λ is the coefficient of heat conductivity.

In the general form for welded product at FSW (Figure 1) during the movement of the center (x_0, y_0, z_0) of working tool (Figure 2) at welding speed v_w the power of heat evolution per a unit of the area of contact between the tool and joint material in an arbitrary point (x, y, z) at moment of time t is described by the following dependence

$$-\lambda\frac{\partial T(x,y,z,t)}{\partial n}=\mu P_n\omega r, \quad (2)$$

- at $z = 0$ and $R_1 < r < R_2$ (on the surface in the tool shoulder zone),
- at $0 < z < \delta$ and $r = R_1$ (by thickness in the tool pin zone),

where μ is the friction coefficient; P_n is the normal force per a unit of area (pressure) in the contact point; ω is the angular speed of the tool rotation; $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$ is the distance of the considered point of contact from the axis of working tool rotation (x_0, y_0); R_1 is the pin radius; R_2 is the shoulder radius; δ is the thickness of the elements being welded. The shoulder angle of inclination β can be ignored, as a small angles ≤ 2–3° the increase of the area of contact in the shoulder zone that does not exceed 5 %.

Boundary conditions on the surfaces of joint elements (Figure 3), taking into account the convective heat exchange with the environment, were assigned as follows:

$$q = -h(T_{out} - T), \quad (3)$$

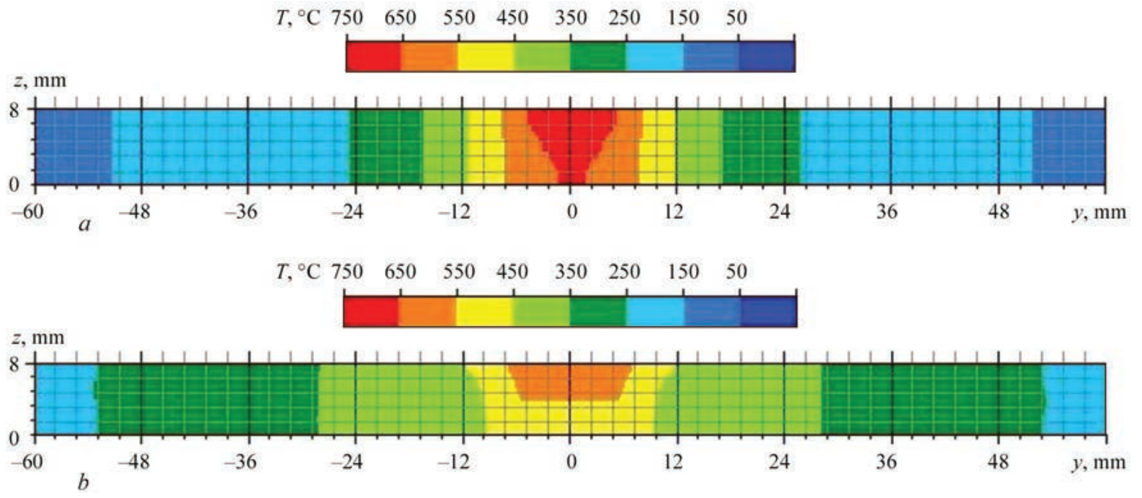

Figure 3. Results of numerical calculation of temperature distribution across the plate thickness in the cross-section in welding butt joints for a model of arc welding (*a*) of 500×500 mm plate, δ = 8 mm and model of FSW (*b*) of a 300×300 mm plate, δ = 8 mm

where T_{out} is the ambient temperature; q is the heat flow; h is the coefficient of heat transfer from the surface at convective heat exchange with the environment (usually under the conditions of natural convection in air T_{out} = 20 °C, h = 10–20 W/(m²·°C).

After establishing the temperature distributions at FSW numerical determination of the stresses and strains is conducted by a similar algorithm, as for arc welding, by successive observation during the occurrence of thermodeformational processes in the joint material from the beginning of heating to complete cooling by elastoplastic analysis and finite elements methods [13]. The difference of the model of stress and strain determination at FSW and model for arc welding is the condition of rigid fastening of the elements during welding and subsequent cooling, i.e. boundary conditions in the fastening zone along the entire length of the elements being welded are found at a short distance ≈ 30–40 mm) from the weld

$$U_x(x, y, z, t) = U_y(x, y, z, t) = U_z(x, y, z, t) = 0 \text{ at } |y| > y_w, \quad (4)$$

where U_x, U_y, U_z are the movements in the longitudinal and transverse directions and across the thickness of the joint elements; y_w is the distance from the welded joint axis to location of the fastening devices.

In the elastoplastic definition the strain tensor can be presented in the following form:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \quad (i, j = x, y, z), \quad (5)$$

where ε_{ij}^{e}, ε_{ij}^{p} is the tensor of elastic and plastic strains, respectively. Components of tensors of stresses σ_{ij} and elastic strains ε_{ij}^{e} are related to each other by Hooke's law:

$$\varepsilon_{ij}^{e} = \frac{\sigma_{ij} - \delta_{ij}\sigma}{2G} + \delta_{ij}(K\sigma + \varphi), \quad (6)$$

where δ_{ij} is the unit tensor ($\delta_{ij} = 0$, if $i \neq j$, $\delta_{ij} = 1$, if $i = j$); $\sigma = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$; $G = \frac{E}{2(1+\nu)}$ is the shear modulus; $K = \frac{1-2\nu}{E}$ is the volumetric compliance, E is the Young's modulus; v is the Poisson's ratio; φ is the function of relative elongations (volumetric changes), caused by temperature change:

$$\varphi = \alpha(T - T_0), \quad (7)$$

where α is the coefficient of relative temperature elongation of the material.

Plastic strains are related to the stressed state by the equation of the theory of plastic nonisothermal flow, associated with Mises yield conditions:

$$d\varepsilon_{ij}^{p} = d\lambda(\sigma_{ij} - \delta_{ij}\sigma) \quad (i, j = x, y, z), \quad (8)$$

where $d\varepsilon_{ij}^{p}$ is the increment of tensor ε_{ij}^{p} at the given moment of time t due to the deformation history, stresses σ_{ij} and temperature T; $d\lambda$ is the scalar function, which is defined by the yield conditions:

$$\begin{gathered} d\lambda = 0, \text{ if } f = \sigma_i^2 - \sigma_y^2(T) < 0 \\ \text{or } f = 0 \text{ at } df < 0; \\ d\lambda = 0, \text{ if f} = 0 \text{ and } df > 0; \\ f > 0 \text{ state is inadmissible,} \end{gathered} \quad (9)$$

where σ_i is the stress intensity

$$\sigma_i = \frac{1}{\sqrt{2}} \times \sqrt{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx} - \sigma_{zz}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + 6\left(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2\right)},$$

$\sigma_i(T)$ is the material yield point at temperature T.

Mechanical and thermophysical properties of AMg6 aluminium alloy [13, 14]

T, °C	E, MPa	σ_y, MPa	ν	$\alpha\cdot10^6$, 1/°C	λ, W(cm·°C)	$c\rho$, J/(cm^3·°C)
20	71440	155	0.324	22.7	1.18	2.40
100	68770	152	0.327	23.4	1.22	2.51
200	64790	149	0.332	24.5	1.27	2.62
300	60330	143	0.337	25.5	1.33	2.73
400	55400	98	0.343	26.6	1.38	2.85
500	49590	70	0.351	27.6	1.43	3.00

Note. σ_y — yield point. Material density ρ = 2640 kg/m^3, melting temperature range T_{sol} = 560 °C, T_{liq} = 640 °C, specific melting heat of fusion Q_{liq} = 390 kJ/kg [14].

In order to derive results for components of residual stresses σ_{ij} and stains ε_{ij}, the process of development of elastoplastic strains should be considered during a period beginning from a certain initial state. Traditionally used for this purpose is the method of successive observation, when for moment t the solution is sought, if the complete solution for moment $(t - \Delta T)$ is known, where Δt is the step of observation of the development of elastoplastic strains, within which it is possible to approximately assume that this development occurs by a fairly simple loading path. In his case, the connection between the end increment of the tensor of strains $\Delta\varepsilon_{ij}$ and tensor of stresses σ_{ij}, according to [13], can be written as:

$$\Delta\varepsilon_{ij} = \psi(\sigma_{ij} - \delta_{ij}\sigma) + \delta_{ij}(K\sigma) - b_{ij}, \qquad (10)$$

where ψ is the function of the material state in point (x, y, z) at moment t,

$$\psi = \frac{1}{2G} \text{ if } f < 0, \psi = \frac{1}{2G} \text{ if } f = 0, \quad f > 0 \text{ state is inadmissible}; \qquad (11)$$

b_{ij} is the tensor function of additional strains, which is determined by increment of $\Delta\psi$ volumetric

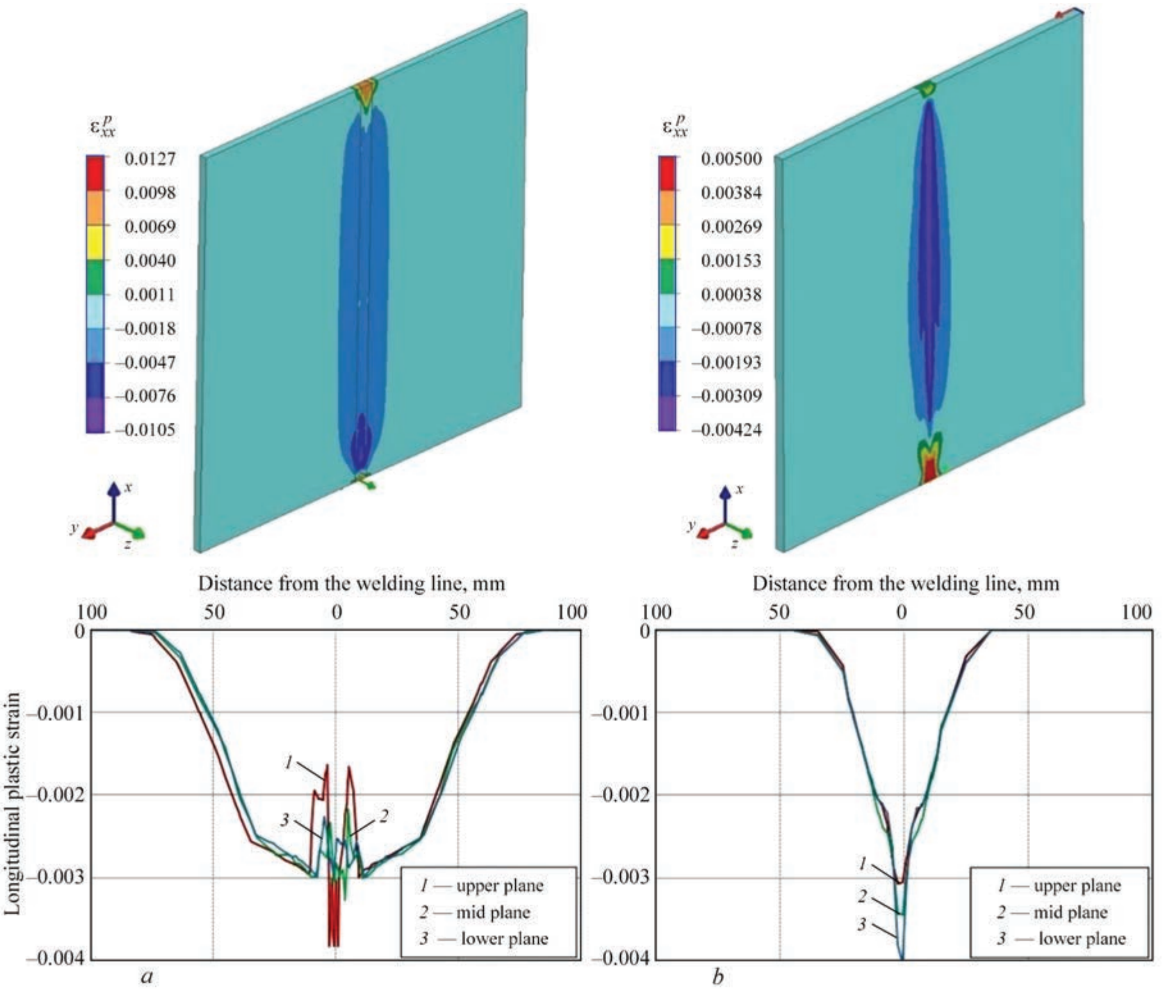

Figure 4. Results of numerical calculation of the distribution of longitudinal plastic strains for welded butt joints in the model of arc welding (*a*) of 500×500 mm plate; δ = 8 mm and model of FSW (*b*) of 300×300 mm plate, δ = 8 mm

changes and known results of the previous stage of observation:

$$b_{ij} = \left[\frac{\sigma_{ij} - \delta_{ij}\sigma}{2G} + \delta_{ij}(K\sigma) \right]_{t-\Delta t} + \delta_{ij}\Delta\varphi \qquad (12)$$

$$(i, j = x, y, z).$$

Yield conditions in the form of (9) include considerable physical nonlinearity in the function of material state ψ, which is usually realized using iteration processes. As a result at each iteration, the physically nonlinear problem becomes a linear problem of the theory of elasticity with a changeable shear modulus, equal to $1/2\psi$, and with additional strains b_{ij}.

MODELING RESULTS

The developed FSW model (1)–(12) was used for calculation of a butt joint of plates of a limited size (300×300 mm, δ = 8 mm, welding mode R_1 = 5 mm, R_2 = 10 mm, ω = 700 rpm, μ = 0.4, P_n = 70 MPa, v_w = 1.7 mm/s) and for presentation of the results of the characteristic distribution of residual stresses and plastic strains, compared to arc welding of a butt joint for a plate of 500×500 mm size, δ = 8 mm (TIG mode: I = 230 A, U = 15 V, v_w = 3 mm/s, efficiency η = 0.6). Mechanical and thermophysical properties of the material, depending on temperature, were taken as those for AMg6 aluminium alloy at modeling (Table).

The results of numerical calculation showed (Figure 3) that the maximum heating temperature at FSW (up to 500–550 °C) is much lower than at arc welding (up to 750 °C and higher) and higher, and it does not reach the melting temperature of the aluminium alloy T_{liq} = 650 °C. The longitudinal component of residual plastic strains at FSW is distributed in a narrower zone than that in arc welding, that is why the integral value of longitudinal shrinkage is approximately 3 times lower (Figure 4). The transverse component of residual plastic strains at FSW is 3 times lower by absolute value than in arc welding (Figure 5). Residual longitudinal stresses at FSW by the maximum value of tensile stresses (up to 150 MPa) are approximately equal to residual longitudinal stresses in arc welding, but the zone of tensile stresses at FSW is much (3 times) narrower (Figure 6). Residual transverse stresses at FSW (up to 14 MPa) are much lower by absolute value than in arc welding (up to 40 MPa) (Figure 7).

Thus, the calculation results obtained using the developed FSW model confirm that undesirable residual

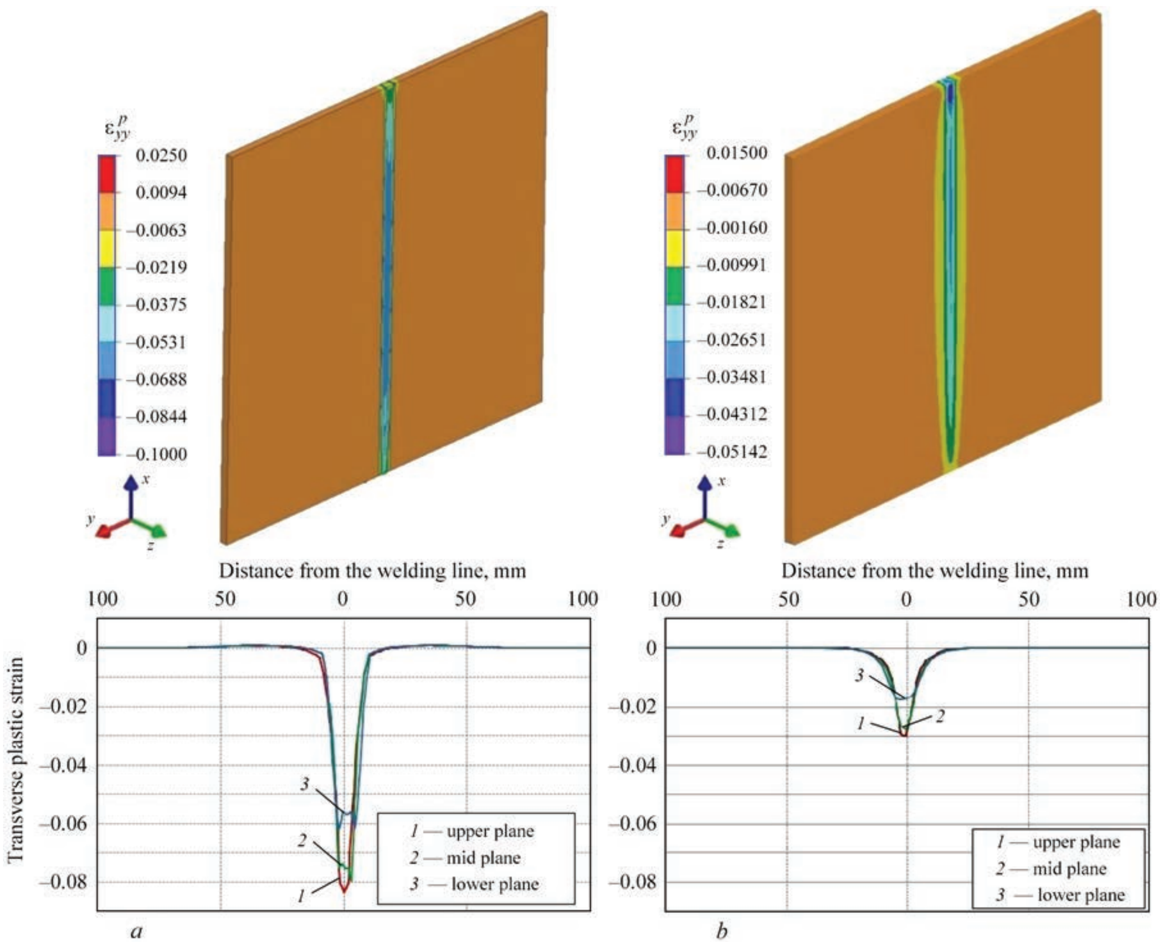

Figure 5. Results of numerical calculation of the distribution of longitudinal plastic strains for welded butt joints in the model of arc welding (*a*) of 500×500 mm plate, δ = 8 mm and model of FSW (*b*) of 300×300 mm plate, δ = 8 mm

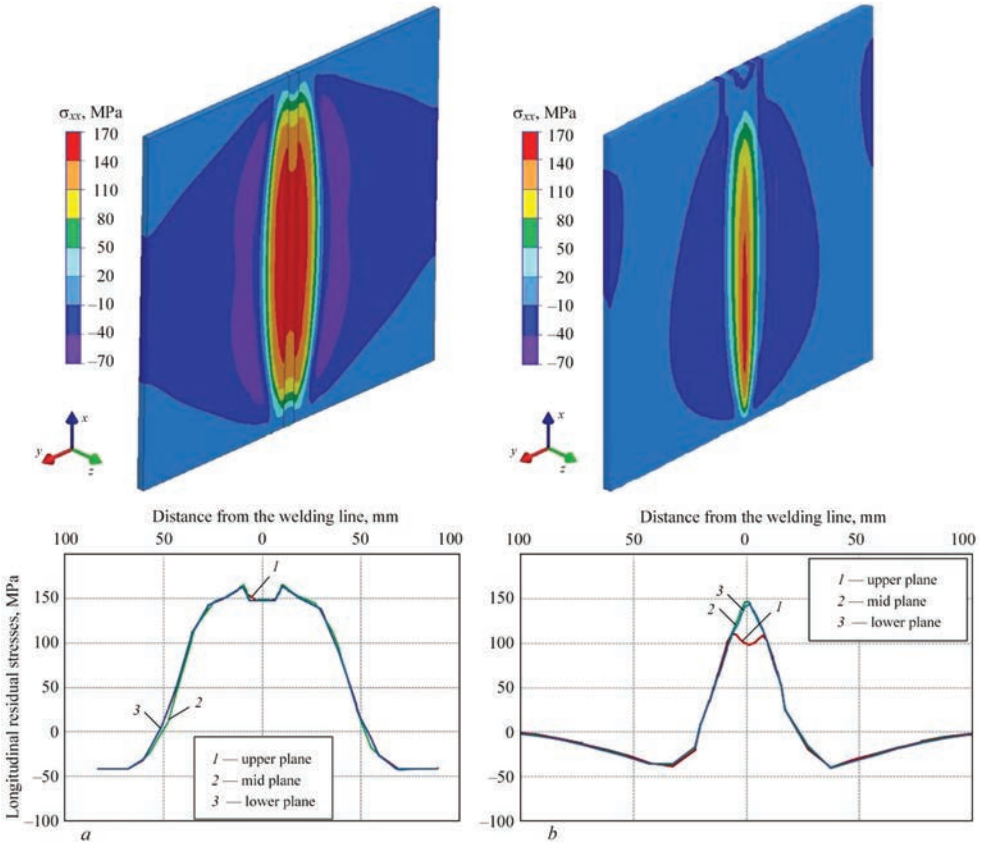

Figure 6. Results of numerical calculation of the distribution of longitudinal plastic stresses for welded butt joints in the model of arc welding (*a*) and model of FSW (*b*)

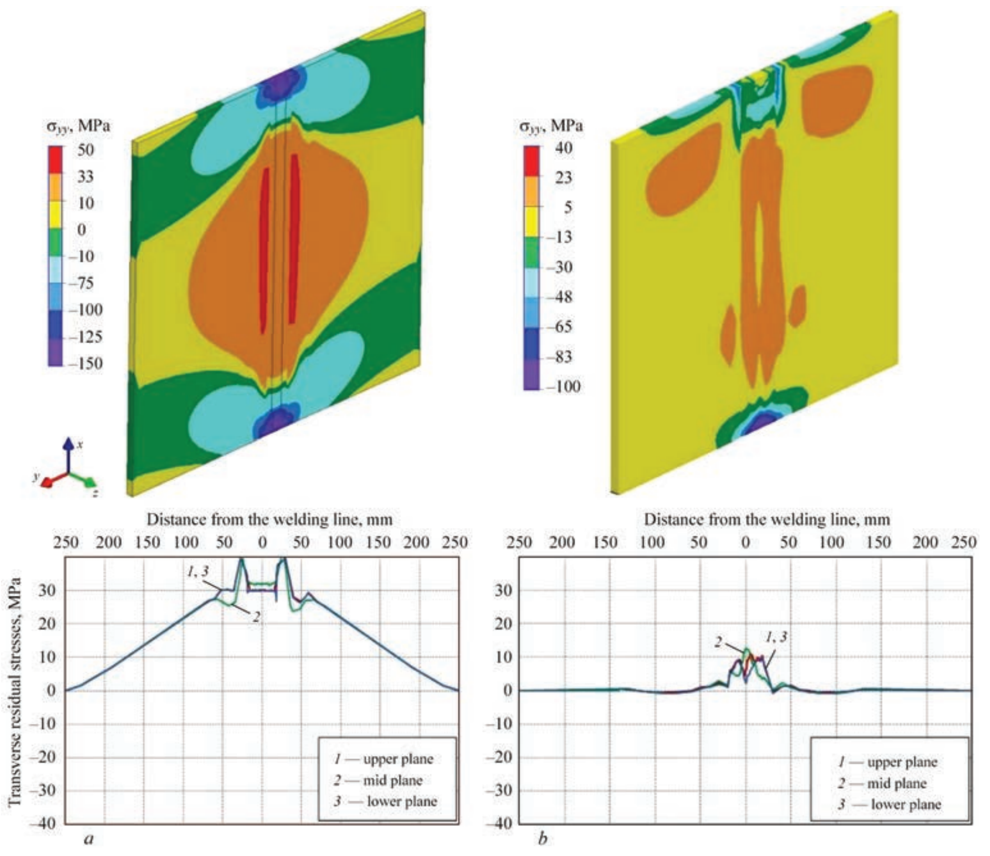

Figure 7. Results of numerical calculation of the distribution of transverse plastic stresses for welded butt joints in the model of arc welding (*a*) and model of FSW (*b*)

stresses and strains form in FSW of aluminium alloys. The residual stresses (particularly the longitudinal component) are close by their magnitude and distribution nature to residual stresses in arc welding, and the level of residual strains is much lower than in the traditional arc welding processes.

More over, there is a whole number of factors, which can influence the prediction accuracy of the developed model. This is, for instance, the dependence of friction coefficient on material temperature, removal of part of the heat into the working tool and fixture for fastening the welded joint elements, as well as partial reduction of evolution of friction heat as a result of material stirring and additional heat evolution from the material plastic strains. Taking these factors into account will, probably, allow more adequate determination of heat evolution in the heat source model, but will greatly increase the complexity of the mathematical model. There is, however, one factor which must be allowed for at FSW modeling in case of aluminium alloys: this is the softening effect, i.e. degradation of mechanical properties of the aluminium alloy in the HAZ. Therefore, at further improvement of the developed model it is rational to analyze consideration of these factors.

CONCLUSIONS

1. A calculation model was developed on the base of the approaches of thermoelastoplastic analysis for numerical determination of residual stresses and strains in the zone of welded butt joints of aluminium alloys produced by FSW. The model has the following main features:

● heat evolution from working tool friction against the joint material;

● conditions of rigid restraint of the joint elements specific to FSW;

● successive observation of the duration of thermodeformational processes in the joint material from the start of heating up to complete cooling.

2. Calculation results obtained from the developed model in welding 8 mm plates from AMg6 alloy confirm the conclusions of the other researchers that undesirable residual stresses and strains form at FSW of aluminium alloys, but their level can be lower than in the traditional arc welding processes. The longitudinal component of residual plastic strains at FSW is close by its magnitude, but it is distributed in a narrower zone, than in arc welding, so that the integral value of the defined longitudinal shrinkage is approximately 3 times lower. The transverse component of residual plastic strains at FSW is three times lower by its magnitude than in arc welding. Residual longitudinal stresses at FSW by the maximum value of tensile stresses (up to 150 MPa) are approximately equal to residual longitudinal stresses in arc welding, but the zone of tensile stresses at FSW is much narrower (3 times). Residual transverse stresses at FSW (up to 140 MPa) are much lower by its absolute value than in arc welding (up to 40 MPa).

3. In order to further increase the prediction accuracy of the developed model it is rational to perform analysis of allowing for such factors as: dependencies of the friction coefficient on material temperature, heat removal into the working tool and fastening fixture, reduction of heat evolution from friction due to material stirring, additional heat evolution due to material plastic strains, as well as degradation of aluminium alloy mechanical properties (softening) at heating during welding.

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CONFLICT OF INTEREST

The Authors declare no conflict of interest

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